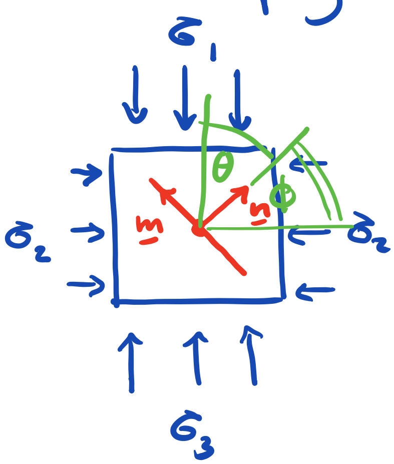
Mohr circle

Mohr circle is a graphical way to display the normal and shear stress on all planes.

For simplicity we look at 2D case, which is already very useful in geology.

Consider physical plane containing σ_1 and σ_3



θ angle between $\underline{n}$ and $\underline{n}_1$

ϕ angle between $\underline{n}$ and $\underline{n}_3$

$$\phi + \theta = \frac{\pi}{2} \quad \Rightarrow \quad \phi = \frac{\pi}{2} - \theta$$

$$\underline{n} = n_1 \underline{n}_1 + n_2 \underline{n}_3$$

$$n_1 = \underline{n} \cdot \underline{n}_1 = |\underline{n}|\,|\underline{n}_1| \cos\theta = \cos\theta$$

$$n_2 = \underline{n} \cdot \underline{n}_2 = \sin\theta$$

$$\Rightarrow \quad \underline{n} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \quad \Rightarrow \quad \underline{m} = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$

Stress in principal frame $\{\underline{n}_i\}$

$$\underline{\underline{\sigma}} = \sigma_1 \, \underline{n}_1 \otimes \underline{n}_1 + \sigma_2 \, \underline{n}_2 \otimes \underline{n}_2 + \sigma_3 \, \underline{n}_3 \otimes n_3$$

traction: $\underline{t}_n = \underline{\underline{\sigma}}\, n = \sigma_1 \cos\theta\, \underline{n}_1 + \sigma_3 \sin\theta\, \underline{n}_3$

normal stress: $\sigma = \underline{n} \cdot \underline{t}_n = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta$

use: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$, $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$\Rightarrow \boxed{\sigma = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2} \cos 2\theta}$$

shear stress: $\tau = \underline{m} \cdot \underline{t}_n = (\sigma_1 - \sigma_3) \sin\theta \cos\theta$

use $2 \sin\theta \cos\theta = \sin 2\theta$

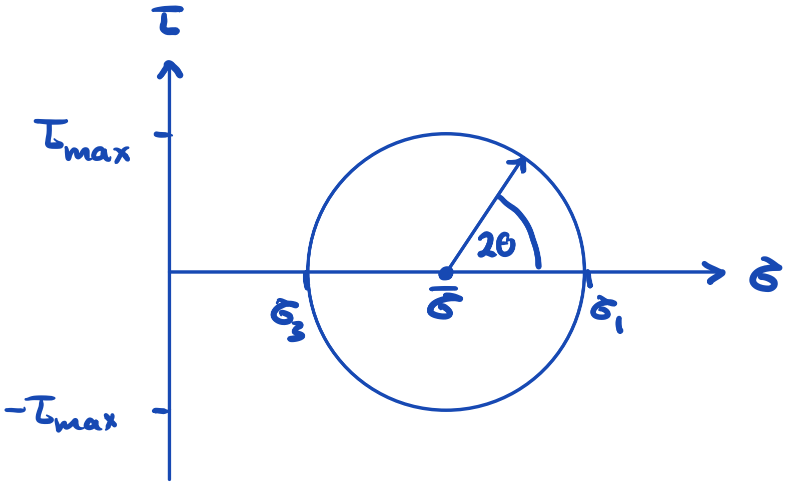
$$\Rightarrow \boxed{\tau = \frac{\sigma_1-\sigma_3}{2} \sin 2\theta}$$

Together these are equations for circle in $\tau\sigma$-space with radius $R = \frac{\sigma_1-\sigma_3}{2}$ and center $\left(\frac{\sigma_1+\sigma_3}{2}, 0\right)$

Note: max shear stress: $\tau_{max} = \frac{\sigma_1-\sigma_3}{2} = R$

mean stress: $\bar{\sigma} = \frac{\sigma_1+\sigma_3}{2}$

For Mohr circle construction compressive stresses are assumed to be positive!



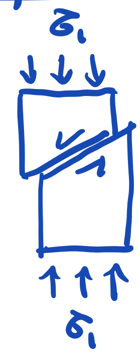
<u>Mohr circle</u>

This is another way of showing that the max. shear stress is at 45° to $\underline{n}_1$ and $\underline{n}_3$.

Reality check: Experimentally observed conjugate fractures are <u>not</u> at 45° !

Failure criteria for shear fracture

Shear fracture is most common type of brittle failure.



Empirical criterion that allows prediction of shear failure.

I) Tresca criterion

Fracture occurs when max. shear stress $\tau_{max} = \tau_{13}$ reaches the shear strength σ_y

$$\boxed{|\tau_{max}| = \frac{\sigma_1 - \sigma_3}{2} = \sigma_y}$$

Note: Failure is not affected by intermediate principal stress and mean stress!

Failure occurs on planes 45° to $\underline{n}_1$

Experiments show angle is smaller than 45°.

II) Coulomb criterion

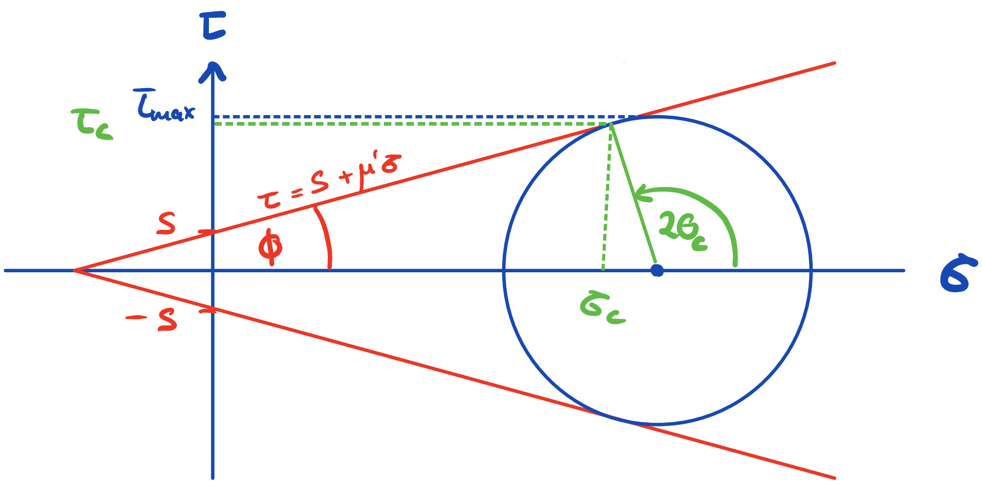
Fracture depends on both mag. of shear stress and the normal stress.

$$\boxed{|\tau| = S + \mu' \sigma}$$

S = cohesive strength ~ 10 – 100 MPa

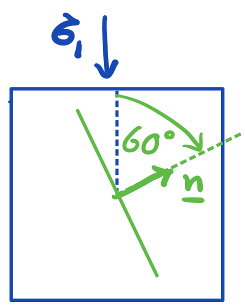
$\mu' = \tan\phi$ internal friction ~ 0.6

$\phi \approx 30°$ angle of internal friction



failure occurs at $\tau_c < \tau_{max}$

angle of failure:



$$\phi + \frac{\pi}{2} + (\pi - 2\theta_c) = \pi$$

$$\theta_c = \underbrace{\frac{\pi}{4} + \frac{\phi}{2}}_{45° + 15°} \approx 60°$$

Byerlee's law

Most brittle rocks already contain pre-existing fractures and fail by reactivating them

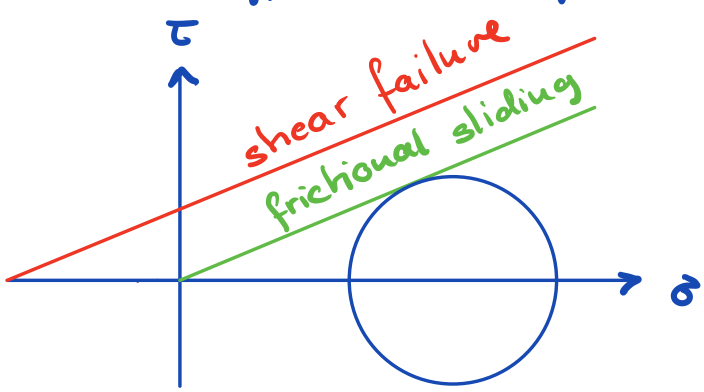
$\Rightarrow$ fail by friction

Criterion for frictional sliding

$$\boxed{|\tau| = S_o + \mu_o \sigma}$$

S_o = cohesion of fault ~ 1 - 10 MPa

μ_o = coefficient of friction ~ 0.5 - 0.8



$S_o \ll S$

Strength of brittle rocks is determined by frictional sliding.