Lecture 11: Stress on a fault

Logistics: - HW 2 is graded ✓ AB^T

some missed second page (1 problem)

- HW 4 due Thu

office hrs today 4-5 pm

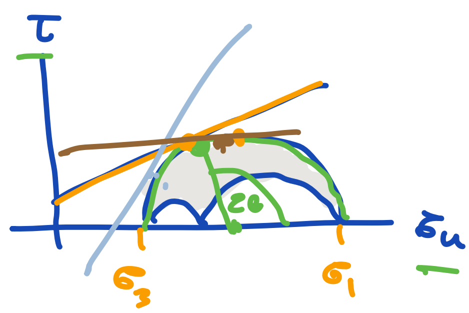
- Problem with 100% copying HW !!!

⇒ assigned lowest grade recieved on HW

Last time: - Mohr circle

- Failure

$|\tau| = S + \mu' \sigma_n$



Today: - Stress on a fault

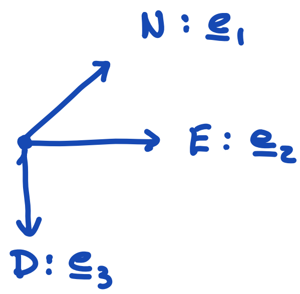
go through all steps with some clarfications

using real example

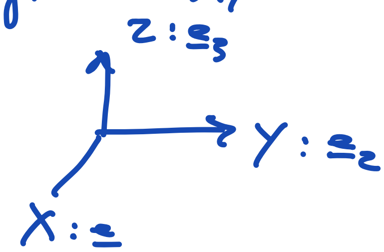
- If time start with tensor calculus !

Fault normals from dip and strike

Geographic coordinate system <u>NED</u>



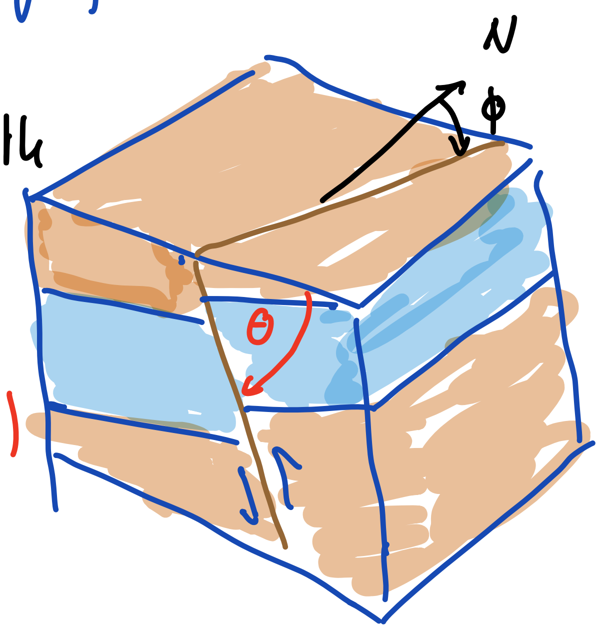
different from XYZ



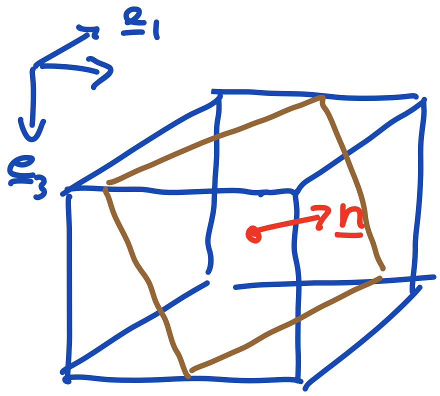
Geological description of fault

strike : ϕ = angle from north

dip : θ = angle from horizontal (perp. from strike)



Q: Given θ, ϕ what is the normal to fault?



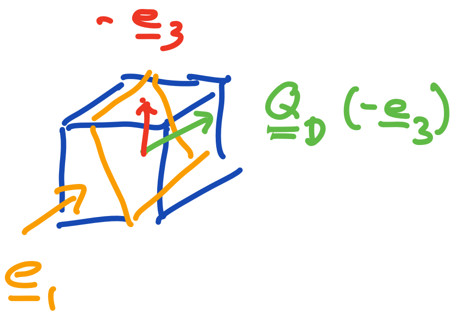
Start with $-\underline{e}_3$

Two rotations:

1) Rotation around $\underline{e}_1$ by dip: $\underline{\underline{Q}}_D = (\underline{e}_1, -\theta)$

$\Rightarrow$ fault with correct dip but strike 0

2) Rotation around $\underline{e}_3$ by strike: $\underline{\underline{Q}}_S = \underline{\underline{Q}}(\underline{e}_3, \phi)$



$$\boxed{\underline{n} = \underline{\underline{Q}}_S \underline{\underline{Q}}_D (-\underline{e}_3)}$$

General rotation matrix

$$\underline{\underline{Q}}(\underline{r}, \theta) = \underline{r} \otimes \underline{r} + \cos\theta \, (\underline{\underline{I}} - \underline{r} \otimes \underline{r}) + \sin \underline{\underline{R}}$$

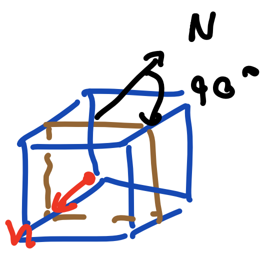
$$\underline{\underline{R}} = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix}$$

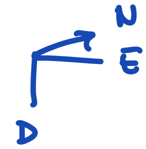
Simple example

dip: $\theta = 90°$ $\left(\frac{\pi}{2}\right)$

strike: $\phi = 90°$ $\left(\frac{\pi}{2}\right)$

by inspection: $\underline{n} = -\underline{e}_1$





$\sin\left(\frac{\pi}{2}\right) = 1 \qquad \cos\left(\frac{\pi}{2}\right) = 0$

Dip rotation:

$$\underline{\underline{Q}}_D = \underline{\underline{Q}}\left(\underline{e}_1, \frac{\pi}{2}\right) = \underline{e}_1 \otimes \underline{e}_1 + \underline{\underline{R}}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

Strike rotation:

$$\underline{\underline{Q}}_S = \underline{\underline{Q}}\left(\underline{e}_3, \frac{\pi}{2}\right) = \underline{e}_3 \otimes \underline{e}_3 + \underline{\underline{R}}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{n} = \underline{\underline{Q}}_S \underline{\underline{Q}}_D (-\underline{e}_3) = \underline{\underline{Q}}_S \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \underline{\underline{Q}}_S \begin{bmatrix} 0 \\ +1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ +1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = -\underline{e}_1$$

$\underline{n}$ is in NED ~~coor~~ frame

Change of basis tensor

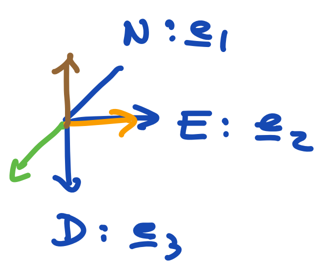
Lecture 8 $\qquad \{\underline{e}_i\}$ & $\{\underline{e}_i'\}$

$\underline{\underline{A}} = A_{ij}\, \underline{e}_i \otimes \underline{e}_j \qquad A_{ij} = \underline{e}_i \cdot \underline{e}_j'$

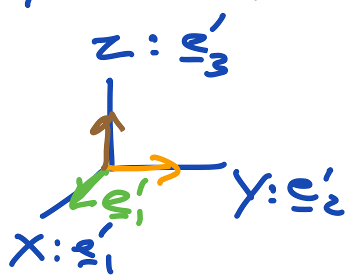
$[\underline{v}] = [A][v]' \qquad [v]' = [\underline{\underline{A}}]^T [\underline{v}]$

$[\underline{\underline{S}}] = [\underline{\underline{A}}][\underline{\underline{S}}]'[\underline{\underline{A}}]^T \qquad [\underline{\underline{S}}]' = [\underline{\underline{A}}]^T [\underline{\underline{S}}][\underline{\underline{A}}]$

NED $\{\underline{e}_i\}$



XYZ $\{\underline{e}_i'\}$



$[\underline{e}_1] = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad [\underline{e}_1']' = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$A_{ij} = \underline{e}_i \cdot \underline{e}_j' \neq [\underline{e}_i] \cdot [\underline{e}_j']'$

$= [\underline{e}_i] \cdot [\underline{e}_j']$

$[\underline{a}] = (\underline{e}_i \cdot \underline{a})\, \underline{e}_i + \ldots \qquad \underline{a} = \underline{e}_1'$

$[\underline{e}_1'] = \underbrace{(\underline{e}_1 \cdot \underline{e}_1')}_{-1} \underline{e}_1 + \underbrace{(\underline{e}_2 \cdot \underline{e}_1')}_{0} \underline{e}_2 + \underbrace{(\underline{e}_3 \cdot \underline{e}_1')}_{0} \underline{e}_3$

$[\underline{e}_1'] = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \qquad [\underline{e}_2'] = [\underline{e}_2] = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$[\underline{e}_3'] = -[\underline{e}_3] = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

Transpose

In general: $\underline{e}_j' = (\underline{e}_i \cdot \underline{e}_j')\,\underline{e}_i = A_{ij}'\,\underline{e}_i$

$$\underline{e}_i = \underset{\uparrow \atop (\underline{e}_i \cdot \underline{e}_j')}{A_{ij}}\,\underline{e}_j'$$

$$\underline{\underline{A}} = \left([\underline{e}_1'], [\underline{e}_2'], [\underline{e}_3']\right) = \begin{array}{c} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ [\underline{e}_1'] & [\underline{e}_2'] & [\underline{e}_3'] \end{array} \end{array}$$

$[\underline{e}_1'] = [A][\underline{e}_1]'$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

Just for plotting: $\underline{\underline{A}}_{xyz}$

$$\underline{n}_{xyz} = \underline{\underline{A}}_{xyz}\,\underline{n}_{ned}$$

Stress tensor in NED frame

typically given $\{\underline{e}'_i\}$ $\quad \underline{e}_i = \underline{v}_i$

principal coord. $\quad \underline{v}_i$ eigen vectors $\quad$ MPa

From data $\quad \sigma_3 =$ vertical ~ 4

$\sigma_2 =$ Shmin ~ 5

$\sigma_1 =$ SHmax ~ 9.5

$$[\underline{\sigma}]' = \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix}$$

$$[\underline{e}'_1]' = [\underline{v}_1]' = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\underline{v}_1 \times v_2 = v_3$

$v_2 \times v_3 = v_1$

$\Rightarrow [\underline{v}_2]$ is given in NED frame (strike)

$[\underline{v}_3]$ vertical (NED)

$[\underline{v}_1] = [\underline{v}_2] \times [\underline{v}_3]$

Change in basis vectors: $\underline{\underline{A}}_\sigma = ([\underline{v}_1], [\underline{v}_2], [\underline{v}_3])$

$$\underline{\underline{S}} = \underline{\underline{V}}\, \underline{\underline{\Lambda}}\, \underline{\underline{V}}^T$$

$\uparrow$

$[\underline{\underline{\sigma}}]'$

$$\Lambda = \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix}$$

$$\underline{V} = \begin{bmatrix} \underline{v}_1 & \underline{v}_2 & \underline{v}_3 \end{bmatrix} = \underline{\underline{A}}$$

$$\underline{\underline{\sigma}}_{ned} = \underline{\underline{A}}_\sigma [\underline{\underline{\sigma}}]' \underline{\underline{A}}_\sigma^T$$