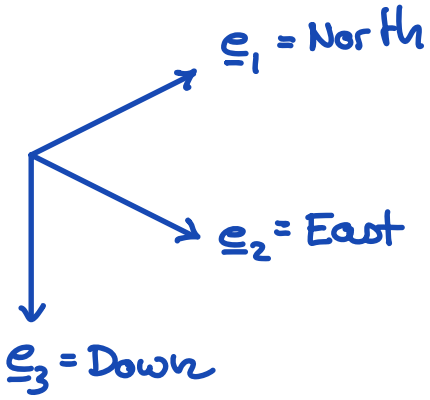
Fault normals from dip and strike

Geographic coordinate system (NED)



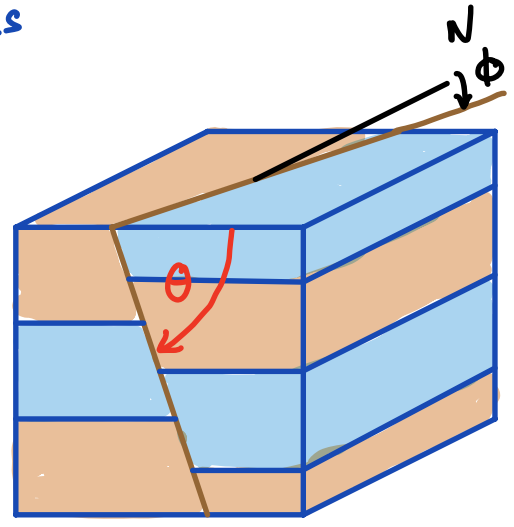
Geological description of a fault

Fault is a discontinuity in the Earth across which rocks have been displaced.

Faults are generated by earthquakes.

Orientation is given by dip and strike angles.

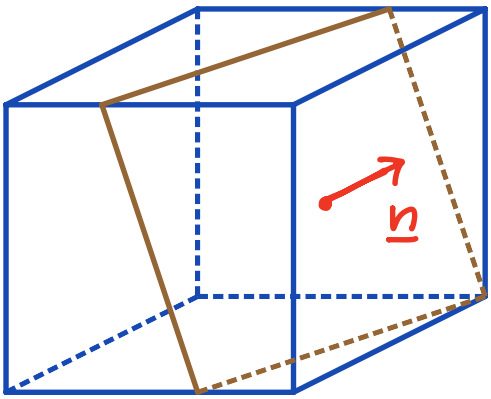
cross-section view



ϕ = strike = angle from north

θ = dip = angle from horizontal (perpendicular from strike)

Q: Given ϕ & θ what is normal to fault plane?



Start with $-\underline{e}_3$

Two rotations:

1) Rotation around $\underline{e}_1$

by dip : $\underline{\underline{Q}}_D = \underline{\underline{Q}}_1(\underline{e}_1, \theta)$

2) Rotation around $\underline{e}_3$

by strike: $\underline{\underline{Q}}_S = \underline{\underline{Q}}(\underline{e}_3, \phi)$

Note, both are counterclock wise rotations !

$$\underline{n} = \underline{\underline{Q}}_S \, \underline{\underline{Q}}_D \, (-\underline{e}_3)$$

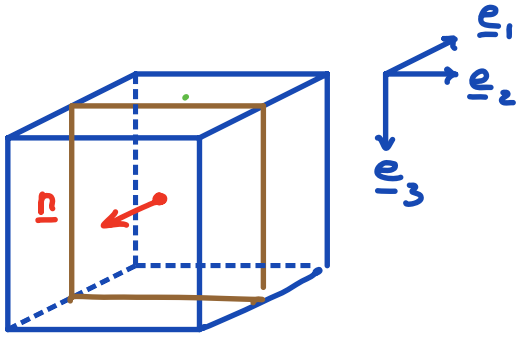
General rotation matrix

$$\underline{\underline{Q}}(\underline{r}, \theta) = \underline{r} \otimes \underline{r} + \cos\theta \, (\underline{\underline{I}} - \underline{r} \otimes \underline{r}) + \sin\theta \, \underline{\underline{R}}$$

Simple example:

dip: $\theta = 90^\circ \; (\frac{\pi}{2})$

strike: $\phi = 90^\circ \; (\frac{\pi}{2})$



by inspection : $\underline{n} = -\underline{e}_1$

$\sin(\frac{\pi}{2}) = 1 \qquad \cos(\frac{\pi}{2}) = 0$

$$\underline{\underline{R}} = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix}$$

Dip rotation:

$$\underline{\underline{Q}}_D = \underline{e}_1 \otimes \underline{e}_1 + \underline{\underline{R}}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\underline{\underline{Q}}_D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

Strike rotation

$$\underline{\underline{Q}}_S = \underline{e}_3 \otimes \underline{e}_3 + \underline{\underline{R}}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{n} = \underline{\underline{Q}}_S \underline{\underline{Q}}_D (-\underline{e}_3)$$

$$= \underline{\underline{Q}}_S \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} = \underline{\underline{Q}}_S \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\underline{n} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = -\underline{e}_1 \checkmark$$