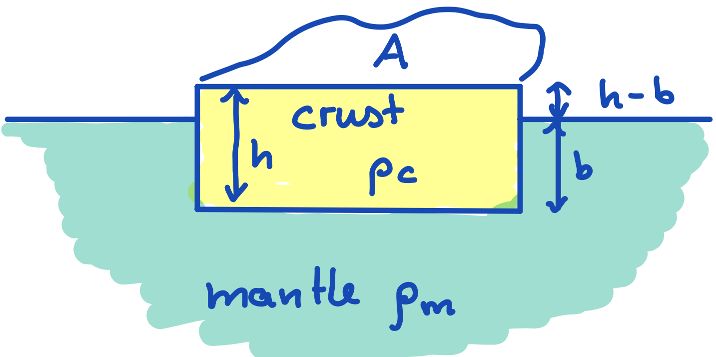
Isostacy

Application of hydrostatic eqbm to Earth.

Consider the continental crust as a rigid block floating in ductile mantle.



h = crustal thickness

b = base of crust

ρ_c = crustal density

ρ_m = mantle density

A = area of crustal block

Q : What is depth of ocean?

mass of crust : $m_c = \rho_c h A$

mass of displaced mantle : $m_m = \rho_m b A$

Hydrostatic force balance:

$$\underline{f} = (m_c - m_m) g = \underline{0} \Rightarrow m_c = m_m$$

$$\rho_c h A = \rho_m b A \Rightarrow \boxed{b = \frac{\rho_c}{\rho_m} h}$$

Depth of the ocean basin:

$$h - b = \left(1 - \frac{\rho_c}{\rho_m}\right) h$$

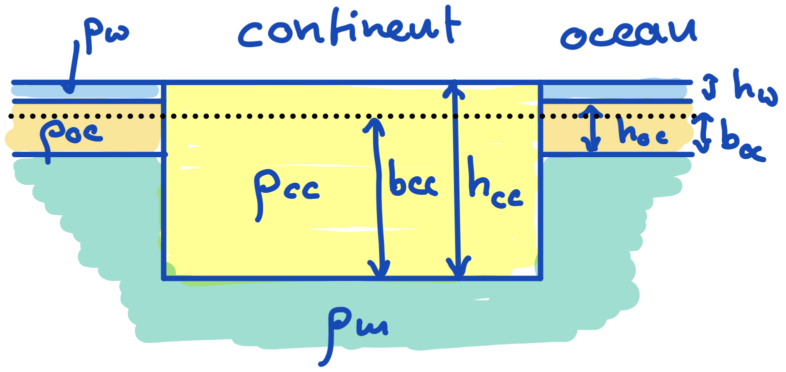
Example: $h \approx 35$ km

$$\left.\begin{aligned} \rho_c &\approx 2750 \ \frac{kg}{m^3} \\ \rho_m &\approx 3500 \ \frac{kg}{m^3} \end{aligned}\right\} \ \frac{\rho_c}{\rho_m} \approx 0.83$$

$\Rightarrow \quad h - b \approx 5.8$ km

The observed depth is closer to 4 km.

More realistic model includes oceanic crust and water column.



$\rho_{oc} \approx 2900 \ \frac{kg}{m^3}$

$h_{oc} \approx 6$ km

$\rho_w \approx 1000 \ \frac{kg}{m^3}$

A_c = area of continent

A_o = area of ocean

Assume that surface of continent is at sea level.

$$\Rightarrow \quad h_{cc} - b_{cc} = h_{oc} + h_w - b_{oc}$$

Hydrostatic force balance of continent:

$$\underline{f} = (m_{cc} - m_m)\, \underline{g} = 0$$

$$\rho_{cc} h_{cc} A_c = \rho_m b_{cc} A_c \qquad b_{cc} = \frac{\rho_{cc}}{\rho_m} h_{cc}$$

Hydrostatic force balance on ocean:

mantle has to support both oceanic crust and water ⇒ two layers

$$\underline{f} = (m_{oc} + m_w - m_m)\, \underline{g} = 0$$

$$\rho_{oc} h_{oc} A_o + \rho_w h_w A_o = \rho_m b_{oc} A_o$$

$$\rho_{oc} h_{oc} + \rho_w h_w = \rho_m b_{oc}$$

3 equations for 3 unknowns

1) $\rho_{cc} h_{cc} = \rho_m b_{cc} \quad \rightarrow \quad b_{cc} = \frac{\rho_{cc}}{\rho_m} h_{cc}$

2) $\rho_{oc} h_{oc} + \rho_w h_w = \rho_m b_{oc}$

3) $h_{cc} - b_{cc} = h_w + h_{oc} - b_{oc}$

solve for depth of the ocean

$$\boxed{h_w = \frac{\rho_{cc} - \rho_m}{\rho_w - \rho_m} h_{cc} - \frac{\rho_{oc} - \rho_m}{\rho_w - \rho_m} h_{oc}} \approx 6.6 \text{ km}$$