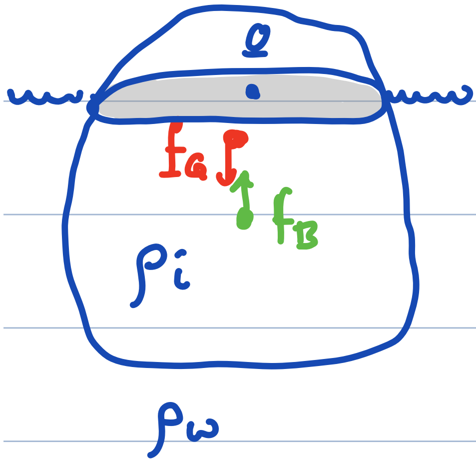
Stability of Ice bergs



floating at surface

x_m above x_V (of displaced fluid)

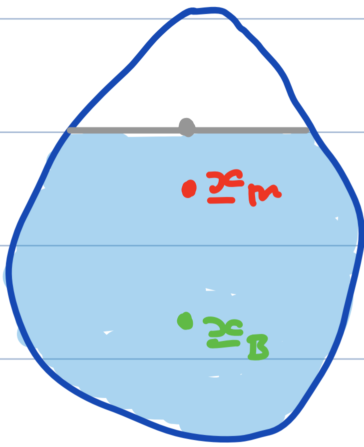
meta stable ?

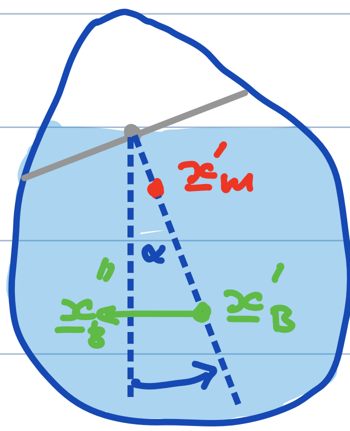
Stability

$\tau_H = m(x_m - x_B) \times g$

x_m = fixed

x_B is not fixed if body rotates !





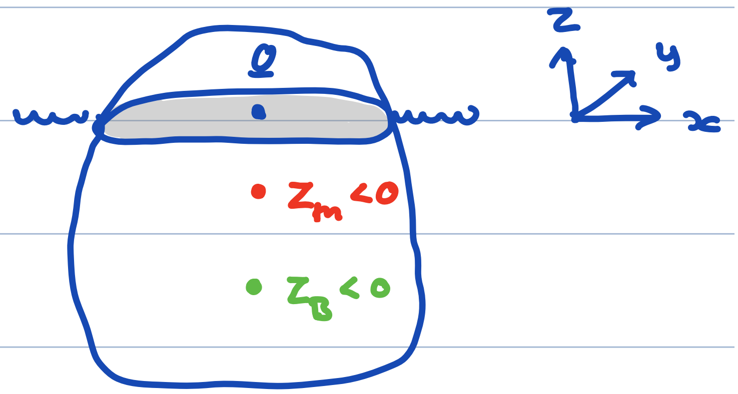
Submerged area changes !

⇒ stabilizes but how much

Center of roll

$$(x_0, y_0) = \frac{1}{A}\int_A (x,y)\,dA$$

$\Rightarrow$ origin



$$\underline{x}_m = \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} \qquad \underline{x}_B = \begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix}$$

initially: $x_m = x_B = 0 \quad y_m = y_B = 0$

Rotation around x-axis

$x_m = x_B = 0$ but y_m & y_B change

$$\underline{\tau} = m(\underline{x}_m - \underline{x}_B) \times \underline{g} \qquad \underline{g} = -g\,\hat{\underline{z}}$$

components:

$$\tau_x = -(y_m - y_B)\,mg$$

$$\tau_y = -(\cancel{x_m - x_B})^{\,0}\,mg$$

$$\tau_z = 0$$

$\Rightarrow$ only torque aroud x-axis

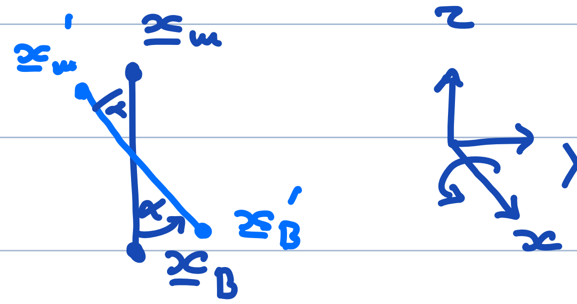
Stability criterium: $\boxed{\tau_x = -(y_m - y_B)\,mg}$

Horizontal shift:

due to rotation:

$$\Delta y_m^r = -\alpha z_m < 0 \quad (z_m > 0)$$

$$\Delta y_B^r = -\alpha z_V > 0 \quad (z_V < 0)$$



change of x_B due to fluid displacement

$$\Delta y_B^F = -\frac{1}{V_D}\int_A (y - y_B)\, u \, dA = -\frac{\alpha}{V_D}\int_A y^2 dA = -\alpha \frac{I}{V_D}$$

$$\boxed{I = \int_A y^2 dA}$$

second moment of waterline

Horizontal change in center of buoyancy

$$\Delta y_B = \Delta y_B^r + \Delta y_B^F$$
$$= -\alpha\left(z_B + \frac{I}{V_D}\right)$$

note $\frac{I}{V_D} \geq 0$

Summary

$$y_m = 0 + \Delta y_m = -\alpha z_m$$

$$y_B = 0 + \Delta y_m = -\alpha\left(z_B + \frac{I}{V}\right)$$

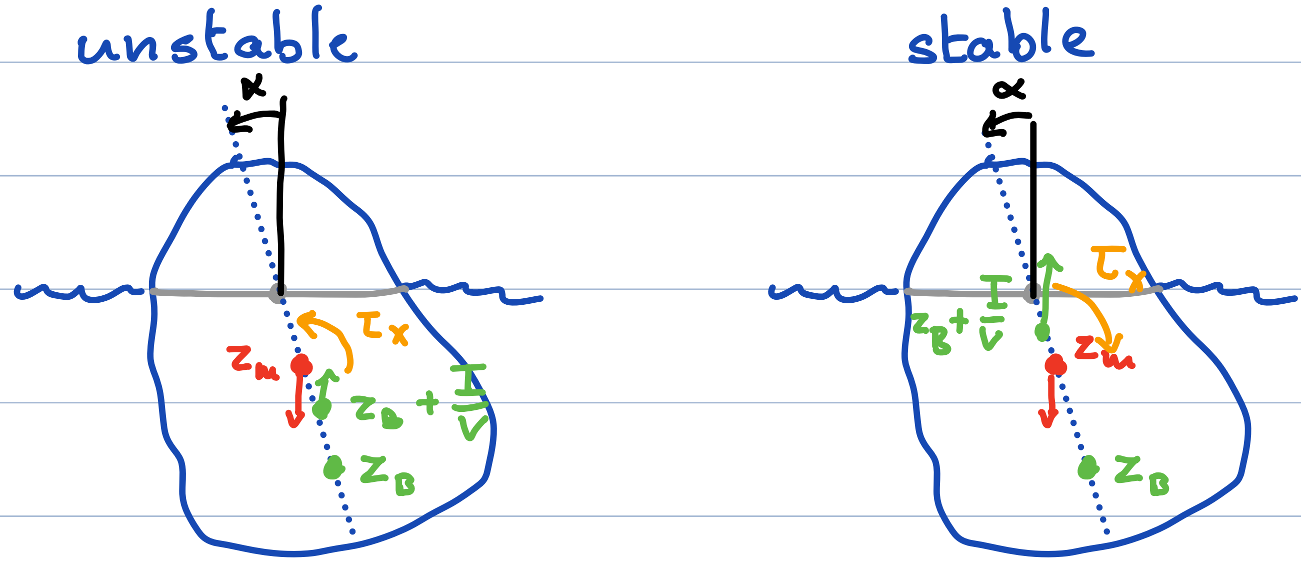
Stability criterium

$$\tau_x = -(y_m - y_B)\, mg$$

$$\boxed{\frac{\tau_x}{mg} = \alpha\left[z_m - \left(z_B + \frac{I}{V_D}\right)\right]}$$

restoring moment: sign of τ_x opposite of α

ice berg is stable if $z_M > z_m$



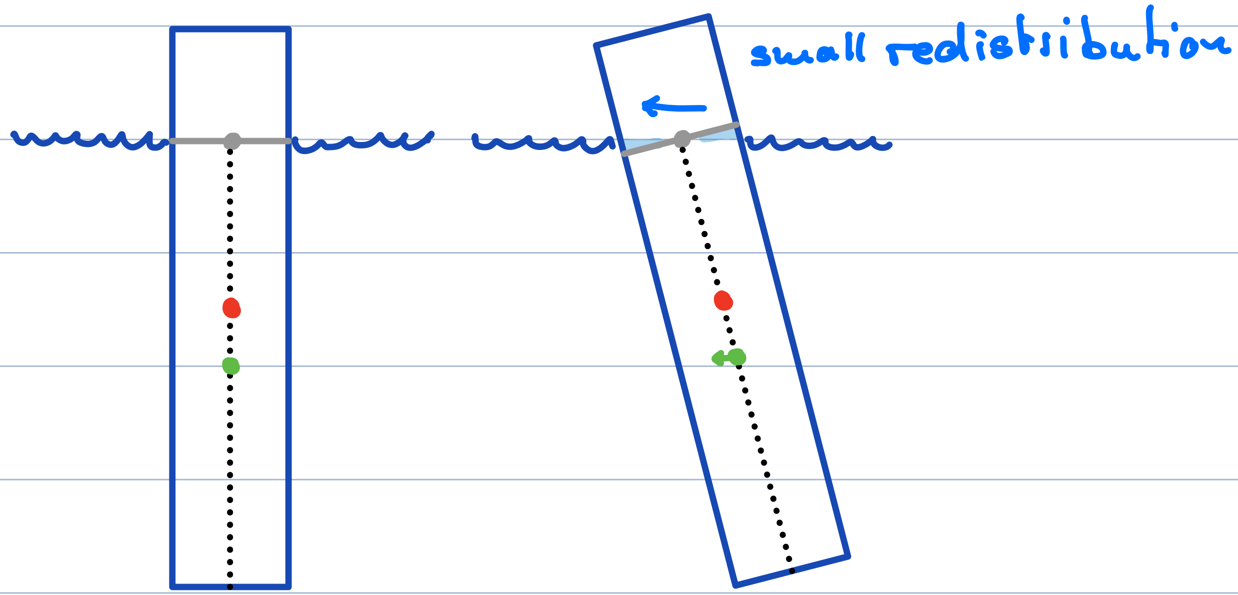
moment <u>same</u> sense of rotation

moment has <u>opposite</u> sense of rotation

General theory for arbitrarily shaped objects and infinitesimal tilt ($\alpha \ll 1$).

Effect of aspect ratio on stability

Tall objects are unstable



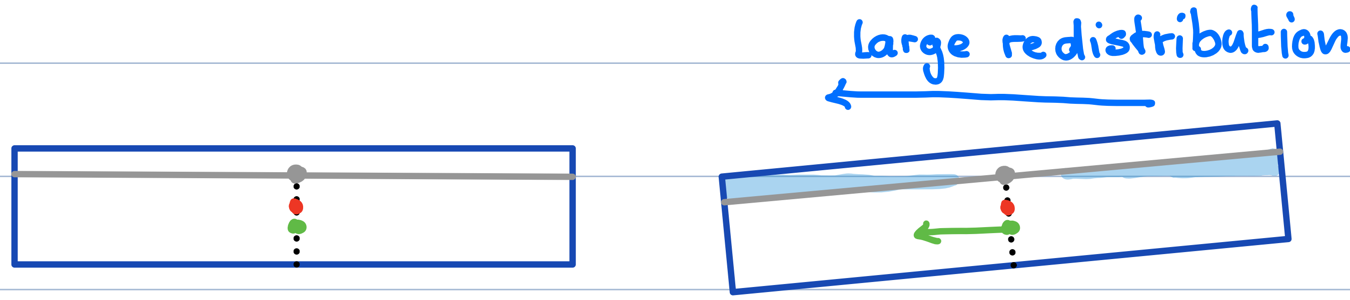
1) z_m and z_B are further apart
$\Rightarrow$ hard to stabilize

2) Long arm leads to a large rotational displacement $\rightarrow$ very destabilizing

3) Redistribution of submerged volume close to centerline (I is small)
$\Rightarrow$ weakly stabilizing

$\Rightarrow$ clearly unstable

Wide objects are stable



1) z_m & z_B are close together
   $\Rightarrow$ easy to stabilize

2) Short arm leads to small rotational displacement $\Rightarrow$ weakly destabilizing

3) Redistribution of submerged volume is far from centerline
   $\rightarrow$ strongly stabilizing

$\Rightarrow$ clearly stable

But where is the boundary?
$\Rightarrow$ HW