Lecture 3: Torque, angular momentum

Logistics: - HW1 due Th 9:30 am

- Tim office hours Wed 9-10 am

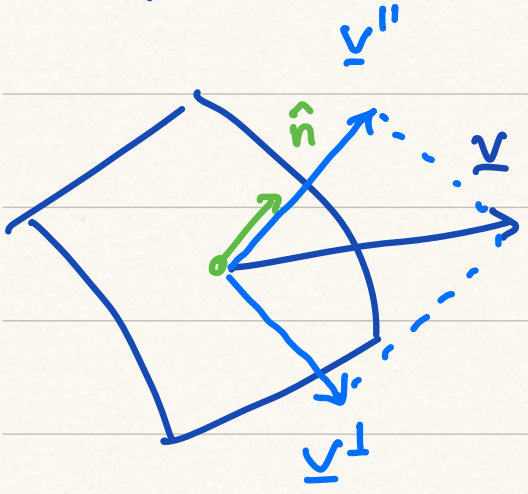
JGB 4.216 B

Triple vector product

$$\underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a} \cdot \underline{c})\,\underline{b} - (\underline{a} \cdot \underline{b})\,\underline{c}$$

$$(\underline{a} \times \underline{b}) \times \underline{c} = (\underline{c} \cdot \underline{a})\,\underline{b} - (\underline{c} \cdot \underline{b})\,\underline{a}$$

=> find normal vector



$$\underline{v}'' = (\underline{v} \cdot \hat{\underline{n}})\,\hat{\underline{n}}$$

$$\underline{v}^{\perp} = \underline{v} - \underline{v}''$$

$$\underline{v}^{\perp} = -(\underline{v} \times \hat{\underline{n}}) \times \hat{\underline{n}}$$

helps practice manipulations important in tensor calculus

Epsilon-Delta Identities

$$\epsilon_{pqs}\epsilon_{nrs} = \delta_{pn}\delta_{qr} - \delta_{pr}\delta_{qn}$$
$$\epsilon_{pqr}\epsilon_{rqs} = 2\delta_{ps}$$

Example: $\underline{a}\times(\underline{b}\times\underline{c}) = (\underline{a}\cdot\underline{c})\underline{b} - (\underline{a}\cdot\underline{b})\underline{c} = \underline{d}$

$\underline{a} = a_q\underline{e}_q \quad \underline{b} = b_i\underline{e}_i \quad \underline{c} = c_j\underline{e}_j \quad \underline{d} = d_p\underline{e}_p$

$$\underline{b}\times\underline{c} = b_i\underline{e}_i \times c_j\underline{e}_j = b_i c_j\ \underline{e}_i\times\underline{e}_j$$
$$= b_i c_j\ \epsilon_{ijk}\underline{e}_k$$

$$\underline{a}\times(\underline{b}\times\underline{c}) = a_q\underline{e}_q \times b_i c_j \epsilon_{ijk}\underline{e}_k$$
$$= \epsilon_{ijk} a_q b_i c_j\ \underline{e}_q\times\underline{e}_k$$
$$= \epsilon_{ijk} a_q b_i c_j\ \epsilon_{qkp}\underline{e}_p$$
$$= \epsilon_{ijk}\epsilon_{qkp}\ a_q b_i c_j\ \underline{e}_p \qquad \epsilon_{qkp} = -\epsilon_{qpk} = \epsilon_{pqk}$$
$$= \epsilon_{ijk}\epsilon_{pqk}\ a_q b_i c_j\ \underline{e}_p$$
$$= (\delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp})\ a_q b_i c_j\ \underline{e}_p$$
$$= \delta_{ip}\delta_{jq}\ a_q b_i c_j\ \underline{e}_p - \delta_{iq}\delta_{jp}\ a_q b_i c_j\ \underline{e}_p$$
$$= a_j b_i c_j\ \underline{e}_i - a_i b_i c_j\ \underline{e}_j$$

$$= \underbrace{a_j c_j}_{\underline{a}\cdot\underline{c}} \underbrace{b_i e_i}_{\underline{b}} - \underbrace{a_i b_i}_{\underline{a}\cdot\underline{b}} \underbrace{c_j e_j}_{\underline{c}}$$

$$= (\underline{a}\cdot\underline{c})\,\underline{b} - (\underline{a}\cdot\underline{b})\,\underline{c} \quad \checkmark$$