

University of Texas at Austin, Jackson School of Geosciences
GEO 325C/398C, 2025
Continuum Mechanics

Midterm Exam – October 7, 2025, 2025 [21 pts]

- (1) [10 pts] **The one with Force (Balance):** Isostasy can explain the formation of basins through extension and thinning of continental crust as shown in Figure 1. Variables with subscript c denote the normal crust and variables with subscript b denote the thinned crust in the basin. The variable h is the total thickness of the crust, z is the elevation the crust rises above the mantle and b the depth the crust is submerged in the mantle. The initial width of the thinned crust is w_0 and the final width of the basin is w . The depth of the basin is $\Delta z = z_c - z_b$. The densities ρ_c and ρ_m of crust and mantle are constant.
- (a) Use area/mass conservation of the area shown in gray to obtain an expression for h_b [2 pts].
- (b) Use the hydrostatic force balance on unstretched crust to obtain an expression for b_c . The gravitational body force, i.e., weight, is given by $f_G = m_c \mathbf{g}$, where m_c is the mass of the crust. The hydrostatic surface force, i.e., buoyancy, is given by $f_B = -m_d \mathbf{g}$, where m_d is the mass of the displaced mantle [2 pts].
- (c) Use the hydrostatic force balance on the stretched crust in the basin to obtain an expression for b_b and use the results in (a) to express it in terms of h_c [2 pts].
- (d) Use the variables derived above to find an expression for the depth of the basin Δz in terms of h_c , ρ_c/ρ_m and w_0/w [3 pts].
- (e) Use the following rounded parameters $\rho_c/\rho_m = 0.9$, $w_0/w = 0.5$ and $h_c = 30$ km to compute Δz using the formula derived above [1 pt].

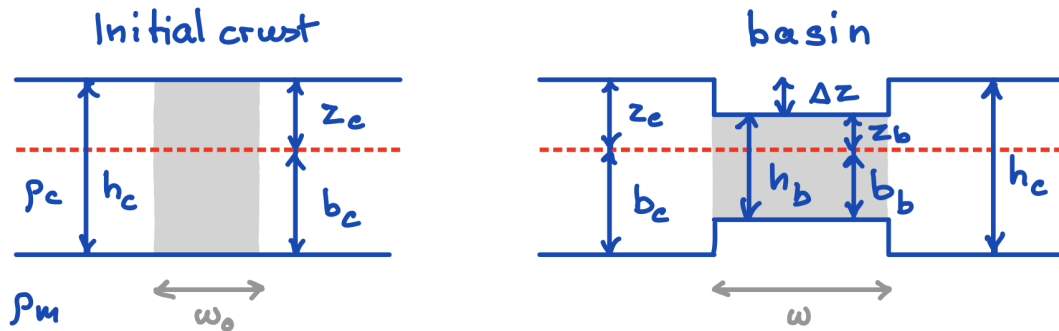


FIGURE 1. Geometry of the isostasy problem for basin formation. The gray area is being stretched and thinned.

- (2) [6 pts] **The One with Rotations:** A general rotation around axis, $\mathbf{r}$, by the clockwise angle, θ , is given by

$$\mathbf{Q}(\hat{\mathbf{r}}, \theta) = \hat{\mathbf{r}} \otimes \hat{\mathbf{r}} + \cos \theta (\mathbf{I} - \hat{\mathbf{r}} \otimes \hat{\mathbf{r}}) + \sin \theta \mathbf{R}(\hat{\mathbf{r}}),$$

where $\mathbf{R}(\hat{\mathbf{r}})$ is the axial tensor

$$\mathbf{R}(\hat{\mathbf{r}}) = \epsilon_{ijk} r_j \mathbf{e}_i \otimes \mathbf{e}_k = \begin{bmatrix} 0 & -\hat{r}_3 & \hat{r}_2 \\ \hat{r}_3 & 0 & -\hat{r}_1 \\ -\hat{r}_2 & \hat{r}_1 & 0 \end{bmatrix}.$$

Answer the questions below for a general rotation, i.e., do not assume a particular axis or angle and do **not** use index notation for this problem.

- (a) Show that the axis of rotation, $\hat{\mathbf{r}}$, remains unchanged by rotation, $\mathbf{Q}\hat{\mathbf{r}} = \hat{\mathbf{r}}$. Use the dyadic property, $(\mathbf{a} \otimes \mathbf{b})\mathbf{c} = (\mathbf{b} \cdot \mathbf{c})\mathbf{a}$ and definition of the axial tensor, $\mathbf{R}(\hat{\mathbf{r}})\mathbf{u} = \hat{\mathbf{r}} \times \mathbf{u}$ [2 pts].
- (b) Show that the transpose of a rotation tensor, $\mathbf{Q}^T(\hat{\mathbf{r}}, \theta)$, is identical to the rotation tensor with the negative angle, $\mathbf{Q}(\hat{\mathbf{r}}, -\theta)$, i.e., $\mathbf{Q}^T(\hat{\mathbf{r}}, \theta) = \mathbf{Q}(\hat{\mathbf{r}}, -\theta)$. Use the transpose of the dyadic product, $(\mathbf{a} \otimes \mathbf{b})^T = \mathbf{b} \otimes \mathbf{a}$, and the skew symmetry of the axial tensor, $\mathbf{R}(\hat{\mathbf{r}})^T = -\mathbf{R}(\hat{\mathbf{r}})$ [4 pts].
- (3) [5 pts] **The One with Index Notation:** As always, follow the basic pattern:
- Define the vectors, in terms of coefficients and basis vectors, i.e., $\mathbf{a} = a_i \mathbf{e}_i$.
 - Substitute these definitions into the expressions.
 - In dot and cross products, pull out the coefficients to the front so that you have products between base vectors ($\mathbf{e}_i$'s).
 - Use the definitions of dot and cross products to evaluate the products between the base vectors: $\mathbf{e}_i \cdot \mathbf{e}_j = \delta_{ij}$ and $\mathbf{e}_i \times \mathbf{e}_j = \epsilon_{ijk} \mathbf{e}_k$.
 - Simplify the expression further and eliminate any δ_{ij} or ϵ_{ijk} .
 - Convert back from index to dyadic notation.

Rewrite the expression below to replace the cross products with dot products. Use index notation and at the end convert the answer back to dyadic notation.

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = \dots,$$

where $\mathbf{a} = a_i \mathbf{e}_i$, $\mathbf{b} = b_j \mathbf{e}_j$, $\mathbf{c} = c_p \mathbf{e}_p$ and $\mathbf{d} = d_q \mathbf{e}_q$. The $\delta\epsilon$ -identity is given by $\epsilon_{ijk}\epsilon_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}$.