University of Texas at Austin, Jackson School of Geosciences
GEO 325C/398C, 2025
Continuum Mechanics

Midterm Exam – October 8, 2024, 2025 [25 pts]

(1) **The One with Index Notation [8 pts]:** The shear stress, $\mathbf{t}^{\perp}$ on a plane with normal, $\mathbf{n}$, can be computed in three ways:

- $\mathbf{t}^{\perp} = \mathbf{t} - (\mathbf{t} \cdot \mathbf{n})\,\mathbf{n}$
- $\mathbf{t}^{\perp} = \mathbf{P}^{\perp}\mathbf{t} = (\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\mathbf{t}$
- $\mathbf{t}^{\perp} = -(\mathbf{t} \times \mathbf{n}) \times \mathbf{n}$

Use index notation to show that $(\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\mathbf{t} = -(\mathbf{t} \times \mathbf{n}) \times \mathbf{n}$.

Use vectors with the following indices: $\mathbf{t} = t_i\mathbf{e}_i$, $\mathbf{n} = n_j\mathbf{e}_j$ and $\mathbf{n} = n_k\mathbf{e}_k$. The identity is given by $\mathbf{I} = \delta_{jk}\,\mathbf{e}_j \otimes \mathbf{e}_k$ the dyadic product $\mathbf{n} \otimes \mathbf{n} = n_j n_k\,\mathbf{e}_j \otimes \mathbf{e}_k$. The frame identities are $\mathbf{e}_i \cdot \mathbf{e}_j = \delta_{ij}$ and $\mathbf{e}_i \times \mathbf{e}_j = \epsilon_{ijk}\mathbf{e}_k$. The $\delta\epsilon$-identity is given by $\epsilon_{ijk}\epsilon_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}$.

(2) **The One with Rotations [total 9 pts]:** In geoscience a fault plane is defined by its strike, ϕ, and its dip angle, θ. The strike is the angle from north and the dip the angle form the horizontal in the direction perpendicular from the strike. The normal to any fault plane can be found from strike and dip by two subsequent rotations, $\mathbf{Q}_S$ and $\mathbf{Q}_D$, of a unit vector pointing upward. In a North-East-Down (NED) reference frame, $\{\mathbf{e}_i\}$, these rotations are defined as $\mathbf{Q}_D = \mathbf{Q}(\mathbf{e}_1, \theta)$ and $\mathbf{Q}_S = \mathbf{Q}(\mathbf{e}_3, \phi)$, where

$$\mathbf{Q}(\mathbf{r}, \vartheta) = \mathbf{r} \otimes \mathbf{r} + \cos(\vartheta)\,(\mathbf{I} - \mathbf{r} \otimes \mathbf{r}) + \sin(\vartheta)\,\mathbf{R}(\mathbf{r})$$

is the Euler representation of a finite rotation tensor and axial tensor is given by

$$\mathbf{R}(\mathbf{r}) = \epsilon_{ijk} r_j\,\mathbf{e}_i \otimes \mathbf{e}_j = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix}$$

For for a fault with strike $\phi = \pi/2$ (90°) and dip $\theta = \pi/2$ (90°) do the following:

(a) Make a 3D sketch of the fault and its normal **[2 pts]**.

(b) Compute $\mathbf{Q}_S$ and $\mathbf{Q}_D$ **[4 pts]**.

(c) Show that the order of rotations matters by computing both $\mathbf{n}_1 = \mathbf{Q}_S\mathbf{Q}_D(-\mathbf{e}_3)$ and $\mathbf{n}_2 = \mathbf{Q}_D\mathbf{Q}_S(-\mathbf{e}_3)$ **[2 pts]**.

(d) Compare the answers with your sketch and determine the correct normal and hence the correct order of rotations **[1 pt]**.

(3) **The One with the Stress [total 8 pts]:** The representation of stress tensor in the NED frame near the SAFOD borehole is approximately

$$\boldsymbol{\sigma} = \begin{bmatrix} 110 & 5 & 0 \\ 5 & 50 & 0 \\ 0 & 0 & 50 \end{bmatrix} \text{MPa}.$$

The representation of the normal to the San Andreas fault in the NED frame near the SAFOD borehole is approximately given by

$$\mathbf{n} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

These values have been modified to give answers with round numbers. Please do not approximate $\sqrt{2}$ as a decimal number.

(a) Compute the traction, $\mathbf{t} = \boldsymbol{\sigma}\,\mathbf{n}$, on the fault **[2 pts]**.

(b) Compute the projection of the traction parallel, $\mathbf{t}^{\parallel}$, and perpendicular, $\mathbf{t}^{\perp}$, to the fault normal, $\mathbf{n}$ **[4 pts]**.

(c) Compute the normal, $\sigma_n = |\mathbf{t}^{\parallel}|$ and shear stress, $\tau = |\mathbf{t}^{\perp}|$, on the San Andreas fault near the SAFOD borehole **[2 pts]**.