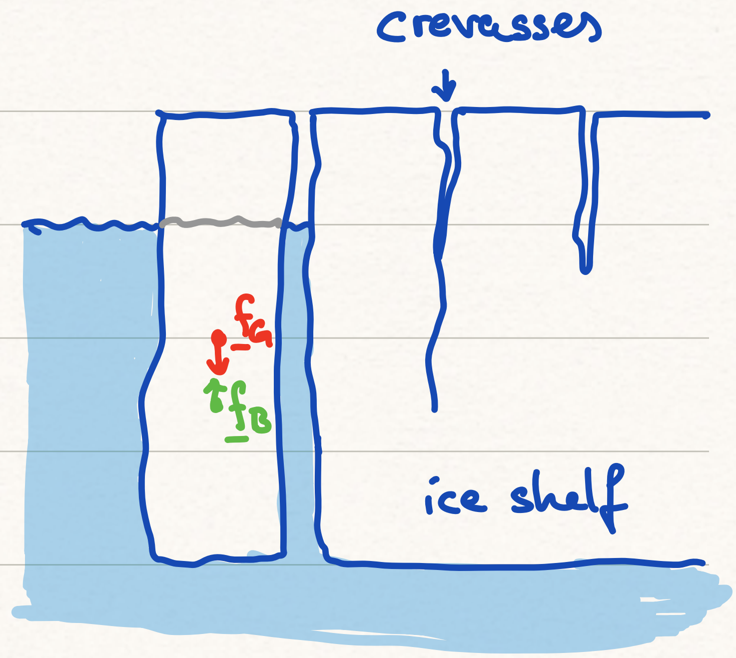
Stability calving fronts

Ice bergs formed by calving are often close to rectangular prisms and unstable !



Rectangular Prism

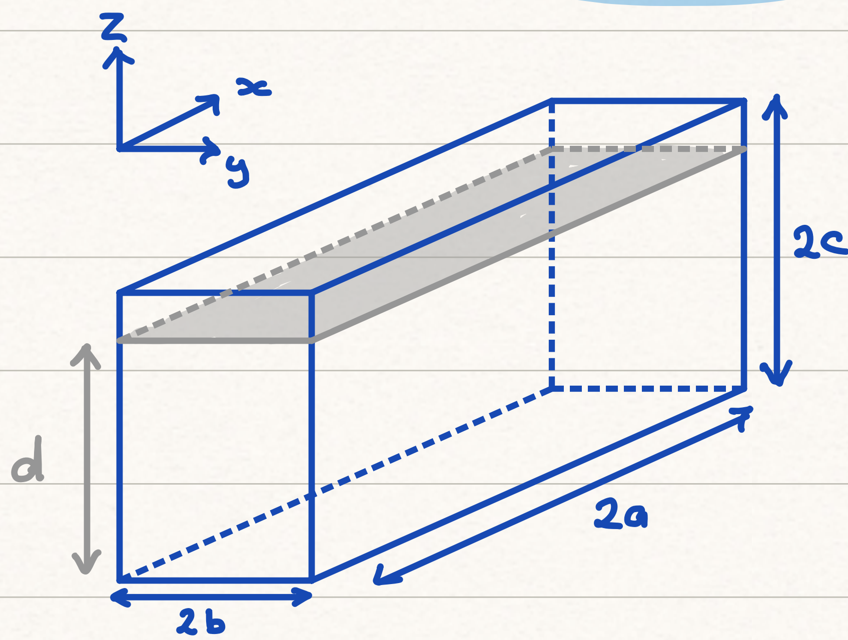
dimension: 2a, 2b, 2c

draft: d

Total volume: $V_T = 8abc$

Displaced volume: $V_D = 4abd$

Waterline area: $A = 4ab$



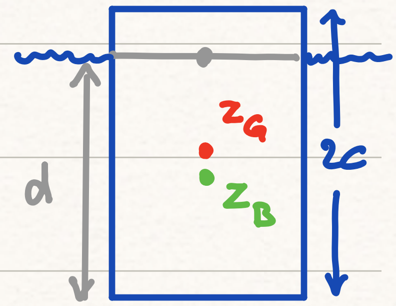
Hydrostatic torque in x-direction

$$\frac{\tau_x}{mg} = \alpha \left[z_G - \left(z_B + \frac{I}{V_D} \right) \right]$$

Moment: $I = \int_{-a}^{a} \int_{-b}^{b} y^2 \, dx \, dy = \frac{4}{3} a b^3$

Initial depth below waterline

$$z_G = c - d < 0 \qquad z_B = -\frac{d}{2} < 0$$



$$I = \frac{4}{3} a b^3 \qquad V_D = 4abd$$

Stable: $$z_G - \underbrace{\left(z_B + \frac{I}{V_D}\right)}_{z_M} < 0 \qquad \Rightarrow \quad z_M > z_G$$

$$z_M = -\frac{d}{2} + \frac{b^2}{3d}$$

Substitute into stability criterion

$$-\frac{d}{2} + \frac{b^2}{3d} > c - d$$

$$\frac{d}{2} - c > -\frac{b^2}{3d}$$

$$d - 2c > -\frac{2}{3}\frac{b^2}{d}$$

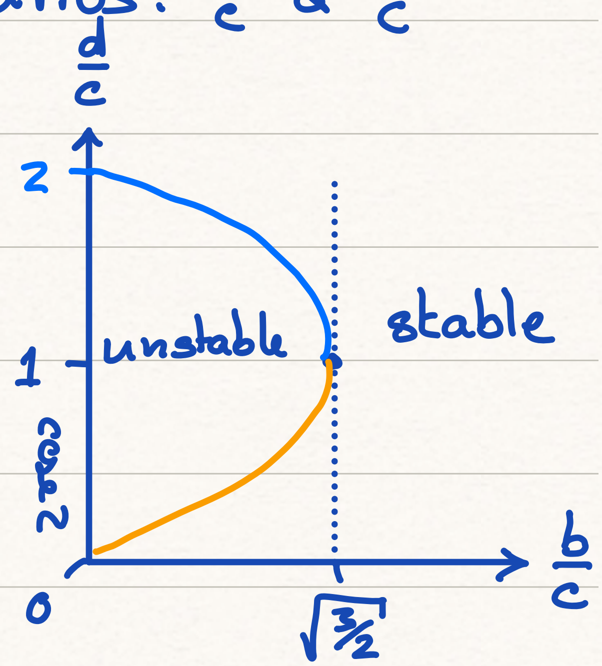
write in terms of aspect ratios: $\frac{b}{c}$ & $\frac{d}{c}$

$$\frac{d^2}{c^2} - 2b > -\frac{2}{3}\frac{b^2}{c^2}$$

complete square $+1$

$$\left(\frac{d}{c}\right)^2 - 2b + 1 > 1 - \frac{2}{3}\frac{b^2}{c^2}$$

$$\boxed{\left(\frac{d}{c} - 1\right)^2 > 1 - \frac{2}{3}\left(\frac{b}{c}\right)^2}$$



l.h.s. is non-negative $(d=c \to 0)$

r.h.s is negative $\Rightarrow$ always stable

$$\frac{2}{3}\left(\frac{b}{c}\right)^2 > 1 \quad \Rightarrow \quad \frac{b}{c} > \sqrt{\frac{3}{2}}$$

Stability boundary for positive r.h.s:

Case 1: $\frac{d}{c} > 1 \Rightarrow \quad \frac{d}{c} > 1 + \sqrt{1 - \frac{2}{3}\left(\frac{b}{c}\right)^2} \Rightarrow$ stable

Case 2: $\frac{d}{c} < 1 \Rightarrow \quad \frac{d}{c} < 1 - \sqrt{1 - \frac{2}{3}\left(\frac{b}{c}\right)^2} \Rightarrow$ stable

Relate $\frac{d}{c}$ to desity contrast usind isostacy

$\rho_i = 917 \frac{kg}{m^3} \qquad \rho_w = 1025 \frac{kg}{m^3}$

$m_i = m_w$

$\rho_i 8 \cancel{ab} c = \rho_w 4 \cancel{ab} d \qquad 2\rho_i c = \rho_w d$

$$\Rightarrow \frac{d}{2c} = \frac{\rho_i}{\rho_w} = 0.89 \Rightarrow 90\% \text{ under water}$$

Solve for stabe aspect ratio $\frac{b}{c}$

$$\left(\frac{d}{c} - 1\right)^2 = 1 - \frac{2}{3}\left(\frac{b}{c}\right)^2$$

$$\frac{2}{3}\left(\frac{b}{c}\right)^2 = 1 - \left(\frac{d}{c} - 1\right)^2$$

$$\left(\frac{b}{c}\right)^2 = \frac{3}{2}\left[1-\left(\frac{d}{c}-1\right)^2\right] =$$

$$= \frac{3}{2}\left[1-\left(\left(\frac{d}{c}\right)^2 - 2\frac{d}{c} + 1\right)\right]$$

$$= \frac{3}{2}\left[\left(\frac{d}{c}\right)^2 + 2\frac{d}{c}\right]$$

$$\boxed{\frac{b}{c} = \sqrt{\frac{3}{2}\left[\left(\frac{d}{c}\right)^2 + 2\frac{d}{c}\right]}}$$

substituting values of ice & water $\frac{d}{c} = 1.8$

$\frac{b}{c} \approx 0.75 = \frac{3}{4}$ $\boxed{\frac{b}{c} \approx \frac{3}{4}}$