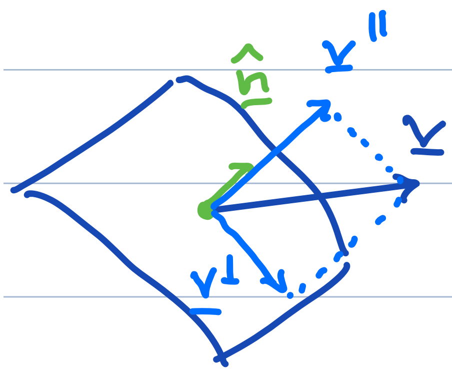
Triple vector product

$\underline{a}, \underline{b}, \underline{c} \in \mathcal{V}$

$$\boxed{\begin{aligned} \underline{a} \times (\underline{b} \times \underline{c}) &= (\underline{a} \cdot \underline{c})\, \underline{b} - (\underline{a} \cdot \underline{b})\, \underline{c} \\ (\underline{a} \times \underline{b}) \times \underline{c} &= (\underline{c} \cdot \underline{a})\, \underline{b} - (\underline{c} \cdot \underline{b})\, \underline{a} \end{aligned}}$$

$\Rightarrow$ find normal component



$$\underline{v} = \underline{v}^{\parallel} + \underline{v}^{\perp}$$

$$\underline{v}^{\parallel} = (\underline{v} \cdot \hat{\underline{n}})\, \hat{\underline{n}}$$

$$\underline{v}^{\perp} = \underline{v} - \underline{v}^{\parallel} =$$

$$\underline{v}^{\perp} = -(\underline{v} \times \hat{\underline{n}}) \times \hat{\underline{n}}$$

Epsilon-delta Identities

In any Cartesian reference frame

$$\boxed{\begin{aligned} \epsilon_{pqs}\, \epsilon_{nrs} &= \delta_{pn}\, \delta_{qr} - \delta_{pr}\, \delta_{qn} \\ \epsilon_{pqs}\, \epsilon_{rqs} &= 2\, \delta_{pr} \end{aligned}}$$

$\Rightarrow$ vector identities with two cross products

Example: $\underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a} \cdot \underline{c})\underline{b} - (\underline{a} \cdot \underline{b})\underline{c} \equiv \underline{d}$

$\underline{a} = a_q \underline{e}_q , \quad \underline{b} = b_i \underline{e}_i , \quad \underline{c} = c_j \underline{e}_j , \quad \underline{d} = d_p \underline{e}_p$

$\underline{b} \times \underline{c} = \epsilon_{ijk} b_i c_j \underline{e}_k$

$$(a_q \underline{e}_q) \times (\epsilon_{ijk} b_i c_j \underline{e}_k) = \epsilon_{ijk} a_q b_i c_j \underbrace{(\underline{e}_q \times \underline{e}_k)}_{\epsilon_{qkp} \underline{e}_p}$$

$$= \epsilon_{ijk} \epsilon_{qkp} a_q b_i c_j \underline{e}_p$$

$$= \epsilon_{ijk} \epsilon_{pqk} a_q b_i c_j \underline{e}_p$$

$$= (\delta_{ip} \delta_{jq} - \delta_{iq} \delta_{jp}) a_q b_i c_j \underline{e}_p$$

$$= a_j b_i c_j \underline{e}_i - a_i b_i c_j \underline{e}_j$$

$$= a_j c_j \; b_i \underline{e}_i - a_i b_i \; c_j \underline{e}_j$$

$$= (\underline{a} \cdot \underline{c}) \underline{b} - (\underline{a} \cdot \underline{b}) \underline{c}$$

Frame Identities

frame $\{\underline{e}_i\}$ = orthonormal basis

$\boxed{\underline{e}_i = \delta_{ij} \underline{e}_j}$ and $\boxed{\underline{e}_i \times \underline{e}_j = \epsilon_{ijk} \underline{e}_k}$