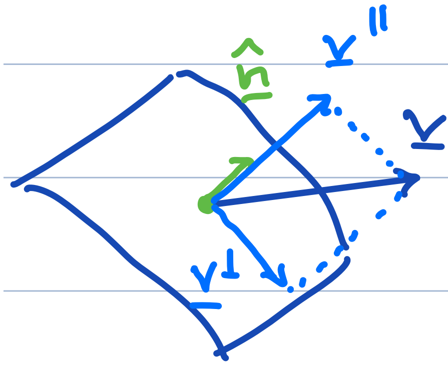
Triple vector product

$\underline{a}, \underline{b}, \underline{c} \in \mathcal{V}$

$$\boxed{\begin{aligned} \underline{a} \times (\underline{b} \times \underline{c}) &= (\underline{a} \cdot \underline{c})\, \underline{b} - (\underline{a} \cdot \underline{b})\, \underline{c} \\ (\underline{a} \times \underline{b}) \times \underline{c} &= (\underline{c} \cdot \underline{a})\, \underline{b} - (\underline{c} \cdot \underline{b})\, \underline{a} \end{aligned}}$$

$\Rightarrow$ find normal component



$$\underline{v} = \underline{v}^{\parallel} + \underline{v}^{\perp}$$

$$\underline{v}^{\parallel} = (\underline{v} \cdot \hat{\underline{n}})\, \hat{\underline{n}}$$

$$\underline{v}^{\perp} = \underline{v} - \underline{v}^{\parallel} =$$

$$\underline{v}^{\perp} = -(\underline{v} \times \hat{\underline{n}}) \times \hat{\underline{n}}$$

Epsilon-delta Identities

In any Cartesian reference frame

$$\boxed{\begin{aligned} \epsilon_{pqs}\, \epsilon_{nrs} &= \delta_{pn}\, \delta_{qr} - \delta_{pr}\, \delta_{qn} \\ \epsilon_{pqs}\, \epsilon_{rqs} &= 2\, \delta_{pr} \end{aligned}}$$

$\Rightarrow$ vector identities with two cross products

Example: $\underline{a}\times(\underline{b}\times\underline{c}) = (\underline{a}\cdot\underline{c})\underline{b} - (\underline{a}\cdot\underline{b})\underline{c}$

$\underline{a}=a_i\underline{e}_i\,,\quad \underline{b}=b_j\underline{e}_j\,,\quad \underline{c}=c_k\underline{e}_k$

$\underline{a}\times(\underline{b}\times\underline{c}) =$ substitute

$a_i\underline{e}_i\times(b_j\underline{e}_j\times c_k\underline{e}_k) =$ pull out coefficients

$a_ib_jc_k\;\underline{e}_i\times(\underbrace{\underline{e}_j\times\underline{e}_k}_{\epsilon_{jkl}\underline{e}_l}) =$ 2nd frame identity

move ϵ to front

$\epsilon_{jkl}\,a_ib_jc_k\,\underbrace{\underline{e}_i\times\underline{e}_l}_{\epsilon_{ilm}\underline{e}_m}$ 2nd frame identity

move ϵ to front

$\epsilon_{jkl}\epsilon_{ilm}\,a_ib_jc_k\,\underline{e}_m$ permute $\epsilon_{ilm}=-\epsilon_{iml}$

$-\epsilon_{jkl}\epsilon_{iml}\,a_ib_jc_k\underline{e}_m$

$-\epsilon_{jkl}\epsilon_{iml}\,a_ib_jc_k\underline{e}_m$ $\delta\epsilon$ identity (one repeated ind.)

$-(\delta_{ji}\delta_{km}-\delta_{jm}\delta_{ki})\,a_ib_jc_k\,\underline{e}_m$

$(\underbrace{\delta_{jm}\delta_{ki}}_{1}-\underbrace{\delta_{ji}\delta_{km}}_{2})\,a_ib_jc_k\,\underline{e}_m$

term 1: $\delta_{jm}\delta_{ki}\,a_ib_jc_k\underline{e}_m$ use transfer prop $m\to j,\ k\to i$

$a_ib_j\,c_i\underline{e}_j$

$$\underbrace{a_i c_i}_{(\underline{a}\cdot\underline{c})} \quad \underbrace{b_j \underline{e}_j}_{\underline{b}}$$

term 2: $-\delta_{ji}\delta_{km}\, a_i b_j c_k \underline{e}_m$ use transfer prop. $j \to i,\ k \to m$

$$-\underbrace{a_i b_i}_{(\underline{a}\cdot\underline{b})} \underbrace{c_m \underline{e}_m}_{\underline{c}}$$

$$\Rightarrow \underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a}\cdot\underline{c})\,\underline{b} - (\underline{a}\cdot\underline{b})\,\underline{c} \quad \checkmark$$