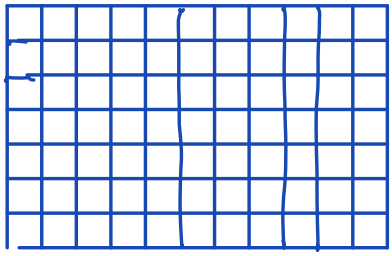
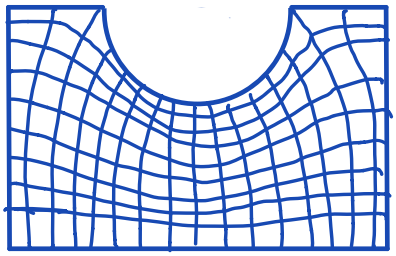
Complex Geometries:

- Discrete operators are for regular cartesian grid



- Non-rectangular domains:

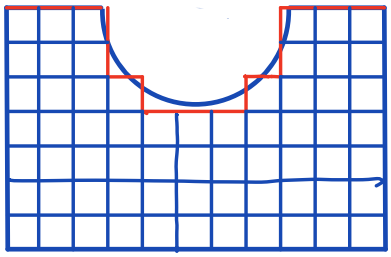
A) Curvilinear mesh



PRO:
- represent geom on relatively coarse mesh
- looks good
- all infra structure for tensor properties

CON:
- limited to relatively simple geometry
- significant complic.
- many numerical pitfalls

B) Embedded boundary



PRO:
- simple to implement
- arbitrarily complex domains

CON:
- need fine mesh
- does not look good

Note: Often people try to achieve this by either setting K very high or very low, depending on BC they want to impose. => Problems: BC's not enforced properly, ill conditioned matrix

Modifying the domain

Step 1: Find cells in crater

$$d = \sqrt{(x-x_0)^2 + (y-y_0)^2}$$

$$dof\_in = Grid.dof(r \leq R_c)$$

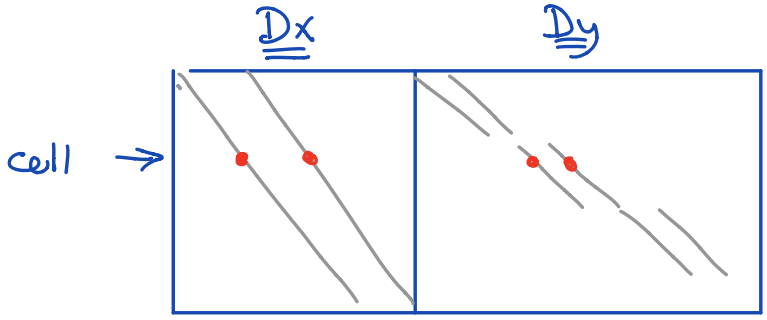
$$dof\_out = Grid.dof(r > R_c)$$

Step 2: Find faces on bud of crater

Given the dof of a cell, what are the dof-f's of the associated faces?

⇒ This info is in the $\underline{\underline{D}}$ matrix

Each row of $\underline{\underline{D}}$ computes the divergence of a cell from the fluxes on its faces.

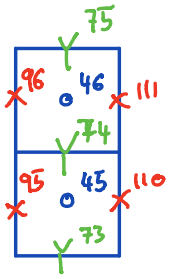


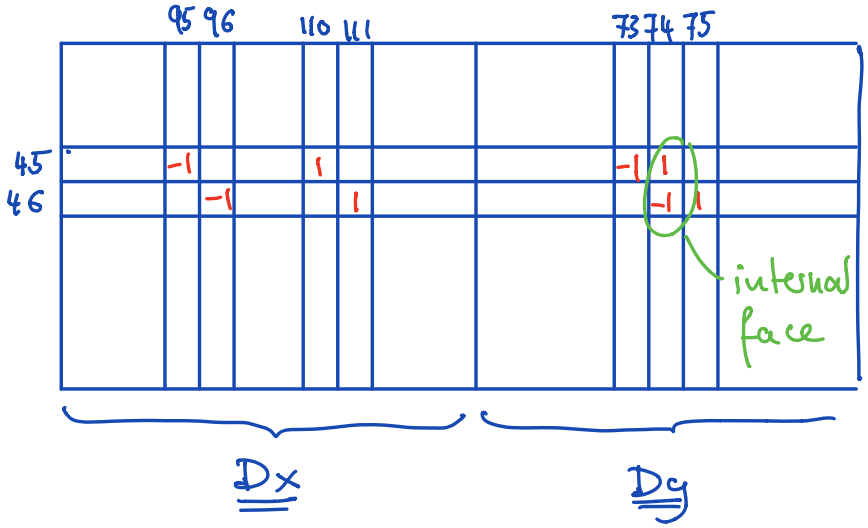
Each row has only 4 non-zero entries corresponding to the 4 faces.

⇒ column indices are the dof-f's of the faces !

But we only want the external faces that form the bud of the crater.

How can we tell if a face is internal to a group of cells or external ?





If two cells share a face the entries will be of the

same magnitude but of opposite sign!

Determining external faces:

1) select all rows of $\underline{\underline{D}}$ corresponding to cells in the crater.

$\underline{\underline{D}}_{in} = \underline{\underline{D}}(dof\_in, :)$;

2) Sum the columns.

sum($\underline{\underline{D}}_{in}$, 1) = 1 by Nf row vector with non-zero entries in position of external faces.

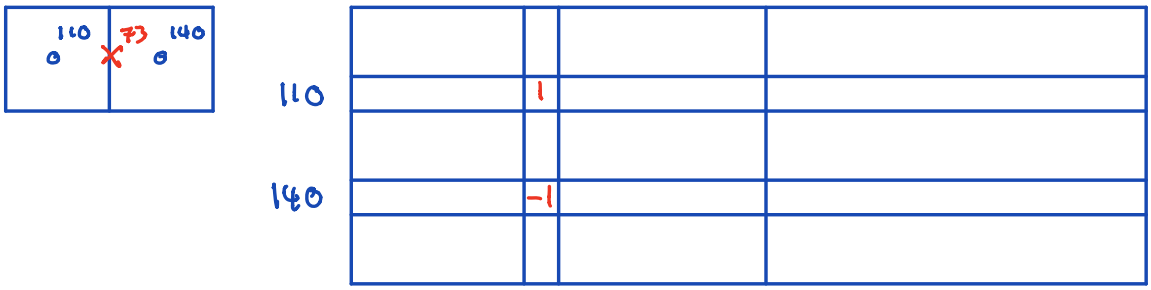
- dof_f_bnd = Grid.dof_f(abs(sum($\underline{\underline{D}}_{in}$, 1)) > ε);

need to use tolerance due to rounding errors.

3) Find the cells aloud the crater boundary

Given a vector of dof_f's what are the associated faces?

Again the information is in $\underline{\underline{D}}$



The columns of $\underline{\underline{D}}$ show which two faces are associated with a face.

To find bnd cells:

1) Select all columns of $\underline{\underline{D}}$ corresponding to dof_f_bnd

$\underline{\underline{Db}} = \underline{\underline{D}}(:, dof\_f\_bnd)$

2) Sum the rows:

sum($\underline{\underline{Db}}$, 2) = N by 1 column vector with non-zero entries in cells along the bnd.

dof_bnd = Grid.dof(abs(sum($\underline{\underline{Db}}$, 2)) > ε)

3) Intersect dof_bnd with dof_out

⇒ boundary cells on outside of crater.