Fluid Mass Balance

General balance eqn $\frac{\partial u}{\partial t} + \nabla \cdot \vec{j}(u) = \hat{f}(u)$

1) Define the unknown to be balanced.

$u \equiv \phi\rho$ $\quad \phi$ = porosity [1]

ρ = pore fluid density $[\frac{M}{L^3}]$

$\Rightarrow u$ is the fluid mass per unit volume of porous medium

Note we assume porous medium is saturated

$\Rightarrow$ entire pore space is filled with fluid

2) Define mass flux of pore fluid

$\vec{j}(u) = j(\phi\rho) \equiv \rho\phi\vec{v} = \rho\vec{q}$ $\quad \vec{v} \equiv$ ave interstitial fluid velocity $[\frac{L}{T}]$

$\vec{q}$ = volumetric flux $[\frac{L^3}{L^2 T} = \frac{L}{T}]$

$\Rightarrow$ fluis mass carried/advected through the porous medium by groundwater flow

3) Volumetric source. $\hat{f} = \rho f$ $\quad \rightarrow$ specify later

Fluid mass balance: $\boxed{\frac{\partial}{\partial t}(\phi\rho) + \nabla \cdot (\rho\vec{q}) = \rho f}$

Constitutive laws. 1) $\vec{q} = -\frac{k}{\mu}(\nabla p + \rho g \hat{z})$ Darcy's law

2) $\rho = \rho(p)$ Equation of state

Here k is the intrinsic permeability $[L^2]$ and μ is the dynamic viscosity $[\frac{M}{LT}]$