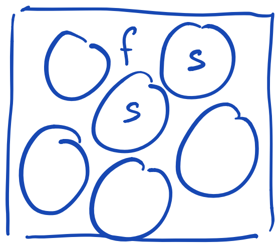
Lecture 1: Intro to porous media

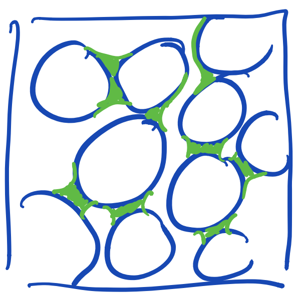


A saturated p. m.

1) solid (s)

2) pore fluid (f)

$\Rightarrow$ single phase flow (linear)



An unsaturated medium

1) Solid phase (s)

2) Wetting phase (w) – water

3) non-wetting phase (n) – gas

$\Rightarrow$ two-phase flow (non-linear)

Volume fractions: $\boxed{\phi_p = \frac{V_p}{V_T}}$ V_p = volume of phase p

$V_T = \sum_p V_p$ total volume

sum over all phases

$\sum_p \phi_p = 1$ vol. frac. constraint.

Porosity: $\phi = \phi_f$ (saturated p.m)

$\phi = \phi_w + \phi_n$ (unsaturated p.m.)

$1 - \phi = \phi_s$ is vol. of solid

Fluid saturations: $s_p = \phi_p / \phi \qquad p \in [w, n]$

s_p is fraction of pore space occupied by p.

ϕ_p is fraction of space occupied by p

$$s_n + s_w = 1$$

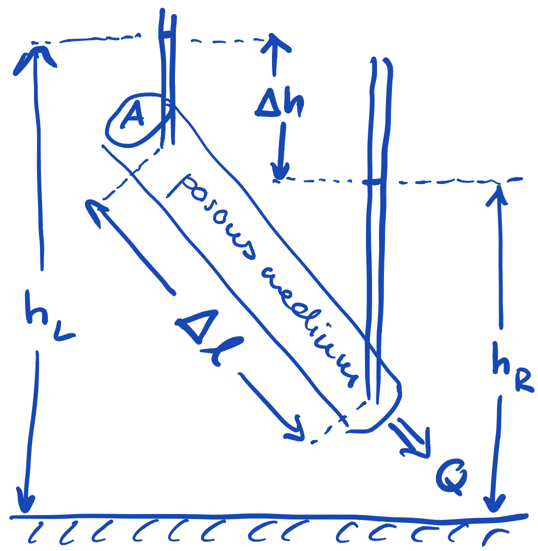
Darcy's law

Basis for all p.m. modeling on the "Darcy scale"

A = ~~cross~~ cross-sec. area $[L^2]$

h_L, h_R = hydraulic heads $[L]$

$\Delta h = h_R - h_L$ change in head $[L]$



Q = vol. flow rate $\left[\frac{L^3}{T}\right]$

ΔL = distance ~~to~~ of head drop

Experimental obs:

1) $Q \sim -\Delta h$

2) $Q \sim 1/\Delta l$

3) $Q \sim A$

$$\left.\begin{array}{l} Q \sim -\Delta h \\ Q \sim 1/\Delta l \\ Q \sim A \end{array}\right\} \quad Q \sim -A \frac{\Delta h}{\Delta l}$$

$\Rightarrow$ Darcy's law : $\boxed{Q = -AK \frac{\Delta h}{\Delta L}}$

$$\underset{\frac{L^3}{T}}{Q} = - \underset{L^2}{A} \, \underset{\frac{L}{T}}{K} \, \underset{\frac{L}{L}}{\frac{\Delta h}{\Delta L}}$$

K = hydralic conductivity [L/T]

constant of proportionality

Observations: 1) Empirical law (o.k.)

2) Macroscopic (good)

3) Q "integrated quantity"

it is over all of A (not good)

For continuum Theory we need a flux!

Rate: amount of something per time $[\frac{\#}{T}]$

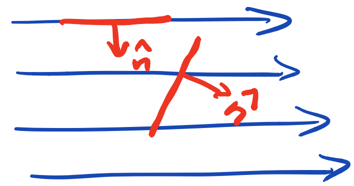
eg. discharge $[\frac{L^3}{T}] \rightarrow$ scalar

Flux: amount of something per area per time

$\left[\frac{\#}{L^2 T}\right] \rightarrow$ vector

eg specific discharge: $q = \frac{Q}{A}\hat{n}$

$\hat{n}$ = unit normal to surface in direction of flow



$\underline{v}$

$q \quad \left[\frac{L^3}{L^2 T} = \frac{L}{T}\right]$ looks like a velocity

but it is **NOT**.

$|q| = -K\frac{\Delta h}{\Delta L}$ $\qquad$ $q = -K\frac{\Delta h}{\Delta n}\hat{n}$

still 1D concept

Darcy's law in 3D

$$\boxed{\underline{q} = -\underline{\underline{K}}\,\nabla h}$$

$\underline{q} = \begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix}$ spec. disch. vect

flux vector

$$\nabla h = \begin{pmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \\ \frac{\partial h}{\partial z} \end{pmatrix}$$ gradient of head

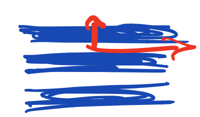
$$\underline{\underline{K}} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{xy} & K_{yy} & K_{yz} \\ K_{xz} & K_{yz} & K_{zz} \end{bmatrix} \qquad \underline{\underline{K}} = \underline{\underline{K}}^T$$

$$\underline{\underline{K}} = K \, \underline{\underline{I}}$$ "isotropic"

homogeneous / heterogeneous

$K = \text{const}$ $\qquad$ $K = K(\underline{x})$

isotropic / anisotropic



$\underline{\underline{K}} = K(\underline{x}) \, \underline{\underline{I}}$ $\qquad$ $\underline{\underline{K}}(\underline{x}) \neq K(\underline{x}) \, \underline{\underline{I}}$

Darcy in terms of p and k

$$q = -\frac{k}{\mu} \left(\nabla p + \rho g \hat{z} \right)$$