Lecture 22: 2D operators on spherical shell

Logistics: - HW 8 5/9

transient non-linear problem → next HW

Last time: - Complex domains

⇒ Embedded boundary

"cut out cells we don't need"

I: Identify inactive cells

II: Find faces on bnd around inactive cells

III: Find cells along the bnd in active domain

3 new arrays: • dof_inactive

• dof_f_bnd faces on bnd

• dof_bnd cells along bnd

Need to find faces of given cells

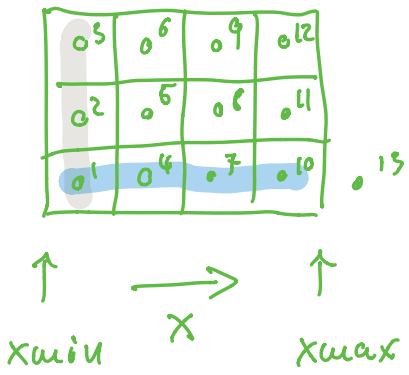
⇒ rows of $\underline{\underline{D}}$

Need to find cells of given faces

⇒ columns of $\underline{\underline{D}}$

Today: 2D spherical shell discretization

HW: $N_x = 4$ $N_y = 3$



Past (1D)

Grid.dof_xmin = 1

Grid.dof_ymin = Nx

2D

Grid.dof xmin = [1, 2, 3]

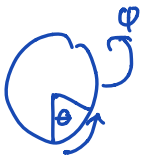
1 : Ny

Grid.dof_ymin = [1 4 7 10]

1 : Ny : N

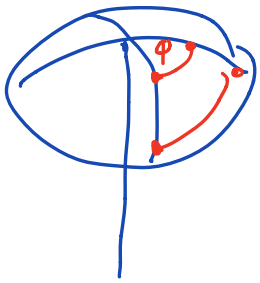
dof_ymax = Ny : Ny : N

2D discrete operators on Spherical shell



Div & Grad on spherical shell

$$\nabla h = \frac{1}{R}\frac{\partial h}{\partial \theta}\hat{\theta} + \frac{1}{R\sin\theta}\frac{\partial h}{\partial \varphi}\hat{\varphi}$$

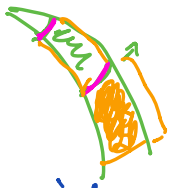


$$\nabla \cdot q = \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, q_\theta) + \frac{1}{R\sin\theta}\frac{\partial q_\varphi}{\partial \varphi}$$

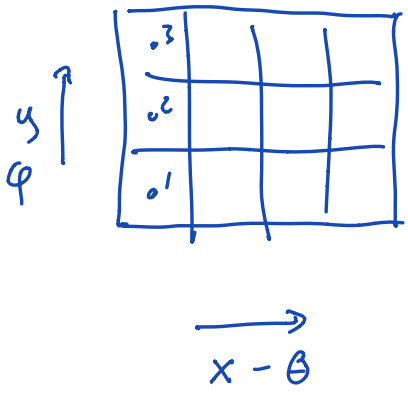
Volume θ faces

Volu

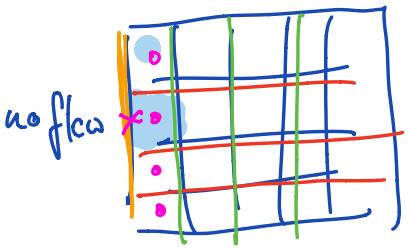
$$q = \begin{pmatrix} q_\theta \\ q_\varphi \end{pmatrix}$$

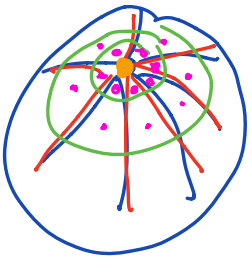


Choose following orientation of grid



⇒ Live script.





Next steps

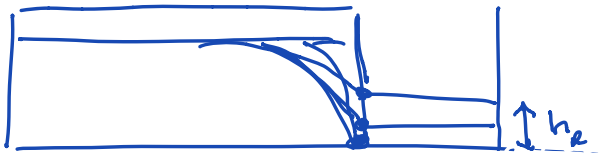
→ unconfined flow

→ cut out "Helles"

→ heterogeneity

→ drainage

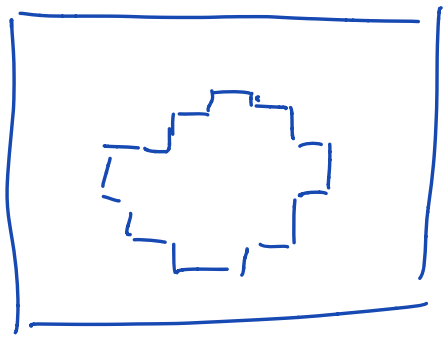
Crater filling



ODE

$$\frac{dh_2}{dt} = \frac{Q}{A h}$$

- Flux on Rnd
  $Q = h \cdot q$
- Mass balance on crater lake
- Newton with ODE



Stream lines

- stream function