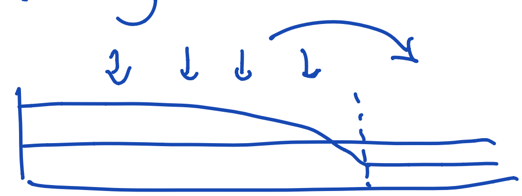
Lecture 24: Filling Craters

Logistics: - Course evaluation
- HW 9 posted
- HW 10 - steady unconfined flow on sphere

Last time: - Steady Ocean-GW coupling



$$-\nabla \cdot kh \nabla h = q_p + q_{pump}$$

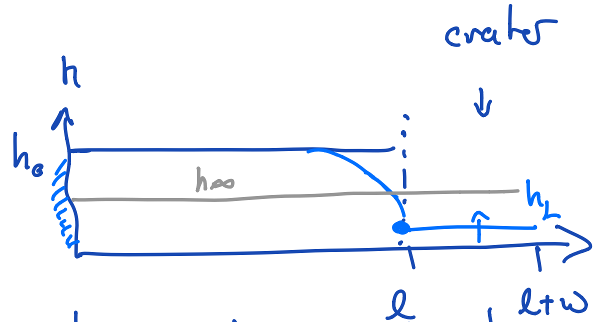
$\Rightarrow$ interesting BC: $h(\ell) \sim \int h \, dx$

Today: Filling - craters

transient

Filling Craters

Crater forms instantaneous



Constraint that water volume is constant.

$$V = V_G + V_L = \phi \int_0^{\ell} h \, dx + w h_L$$

solve for $h_L = \frac{V}{w} - \frac{\phi}{w} \int_0^{\ell} h \, dx$

Steady state head: $h = h_L \equiv h^{\infty}$

$$V = \phi h_\infty \ell + w h_\infty = (\phi \ell + w) h_\infty$$

$$h_\infty = \frac{V}{\phi L + w} = \frac{\phi L h_0}{\phi L + w}$$

$$V = h_0 \ell \phi$$

We have the following transient problem:

PDE: $\phi \frac{\partial h}{\partial t} - \nabla \cdot [K h \nabla h] = 0 \qquad x \in [0, L]$

BC's: $q \cdot \hat{n}|_0 = 0 \qquad h(\ell) = \frac{V}{w} - \frac{\phi}{w} \int_0^{\ell} h \, dx$

IC: $h(x, t=0) = h_0$

⇒ Coupled Dir BC is similar to the last problem

Non-dimensionalize

$$x' = \frac{x}{\ell} \qquad h' = \frac{h}{h_0} \qquad t' = \frac{t}{t_c}$$

substitute into PDE:

$$\frac{\phi h_0}{t_c}\frac{\partial h'}{\partial t'} - \frac{K h_0^3}{\ell^2}\nabla' \cdot h' \nabla' h' = 0 \qquad x' \in [0,1]$$

$$\frac{\partial h'}{\partial t'} - \frac{K h_0 t_c}{\phi l^2}\nabla' \cdot h' \nabla' h' = 0 \quad \Rightarrow \quad \boxed{t_c = \frac{\phi l^2}{K h_0}}$$

substitute into BC:

$$\cancel{h_0}\, h'(1) = \frac{\phi L \cancel{h_0}}{\omega} - \frac{\phi}{\omega}\cancel{h_0}\,\ell \int_0^1 h' \, dx'$$

$$h'(1) = \underbrace{\frac{\phi L}{\omega}}_{\frac{1}{Ca}} \left(1 - \int_0^1 h' \, dx'\right)$$

<u>Dimensionless problem</u>

PDE: $\frac{\partial h'}{\partial t'} - \nabla' \cdot h' \nabla' h' = 0 \qquad x' \in [0,1]$

BC's: $q' \cdot \hat{n}|_0 = 0 \qquad h'(1) = \frac{1}{Ca}\left(1 - \int_0^1 h' \, dx'\right)$

IC: $h'(x', 0) = 1$

$\Rightarrow$ first solve steady problem from last time

$$-\nabla \cdot h' \nabla h' = Pr$$

$$q \cdot \hat{n}\big|_0 = 0 \qquad h'(1) = 1 + \frac{1}{Ca}\left(1 - \int h' dx'\right)$$

Newton-Raphson with new BC

$$r_{N_x}(\underline{h}) = 1 + \frac{1}{ca}\left(1 - \underline{h} \cdot Grid.V\right) - \overbrace{\underline{\underline{B}}\,\underline{h}}^{h_{N_x}}$$

$$\frac{d}{d\varepsilon} r_{N_x}(\bar{h} + \epsilon \hat{h})\Big|_{\varepsilon = 0} = \cancel{1} + \cancel{\frac{1}{ca}} - \frac{1}{ca}(\cancel{\underline{\bar{h}}} + \epsilon \underline{\hat{h}}) \cdot Grid.V - \overbrace{\underline{\underline{B}} \cdot (\cancel{\underline{\bar{h}}} + \epsilon \underline{\hat{h}})}^{h_{N_x}}$$

$$= -\frac{1}{ca}\, Grid.\underline{V} \cdot \underline{\hat{h}} - \underset{\underset{\underline{b}}{\uparrow}}{\underline{\underline{B}}}\, \underline{\hat{h}}$$

because B is vector

$$= -\underbrace{\left(\frac{1}{ca}\, Grid.\underline{V} - \underline{b}\right)}_{\mathbb{J}(N_x, :)} \cdot \underline{\hat{h}}$$