# Energy / Enthalpy conservation equation

For now we ignore that multiple minerals are present and that melting occurs. Initially we simply consider heat conduction in ice.

$\Rightarrow$ single phase energy conservation

General balance equation:

$$\frac{\partial u}{\partial t} + \nabla \cdot \underline{j}(\underline{u}) = \hat{f}_s$$

$u$ = unknown to be balanced $\left[\frac{\#}{L^3}\right]$

$\underline{j}$ = flux $\left[\frac{\#}{L^2 T}\right]$

$\hat{f}_s$ = volumetric source $\left[\frac{\#}{L^3 T}\right]$

1) Conserved quantity

Internal energy / enthalpy : $U_{V,P} = H_P$ $\left[\frac{ML^2}{T^2}\right]$ (Joule)

$u = \frac{H}{m} = h$ specific energy/enthalpy or energy density $\left[\frac{L^2}{T^2}\right]$ $\frac{J}{kg}$

$dh = c_p \, dT$ $\quad c_p = \frac{C_P}{m}$ specific heat capacity at const. p

assuming $m = \rho V$ is constant

$$\frac{\partial H}{\partial t} = \int_\Omega \frac{\partial}{\partial t}(\rho h)\, dV = \int_\Omega \rho \frac{\partial h}{\partial t} dV = \int_\Omega \rho c_p \frac{\partial T}{\partial t} dV \quad \Rightarrow \quad \frac{\partial u}{\partial t} = \rho c_p \frac{\partial T}{\partial t}$$

2) Conductive flux : $\underline{j} = -\kappa \nabla T$

3) Source / Sink : $\hat{f}_s = 0$ for our application

Heat equation : $$\boxed{\rho c_p \frac{\partial T}{\partial t} - \nabla \cdot (\kappa \nabla T) = 0}$$

Note :
- only valid for an incompressible single phase medium at const. pressure
- non-linear because $c_p = c_p(T)$ and $\kappa = \kappa(T)$

# Thermophysical relations for ice

heat capacity: $c_p = 185 + 7.037\,T$ $\quad 1200 - 2100 \quad \frac{J}{kg\,K}$

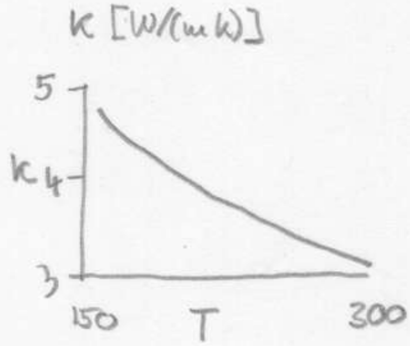
thermal conductivity: $k = 0.4685 + 488.12/T \quad 2.3 - 3.8 \quad \frac{W}{m\,K}$

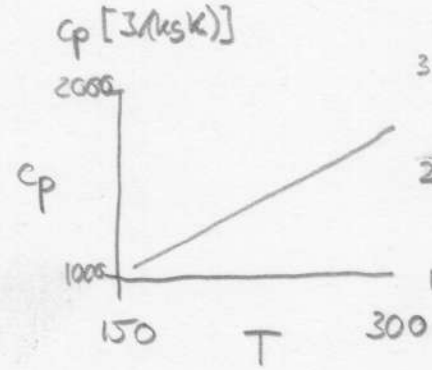
density: $\rho = 917 \ \frac{kg}{m^3}$

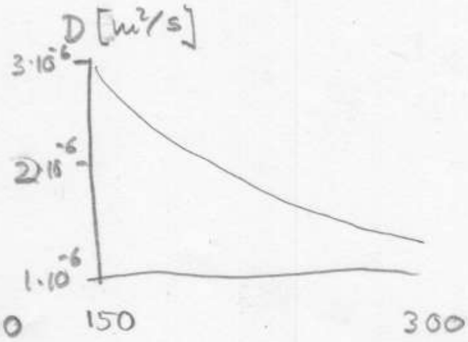
If $k =$ const:

$$\rho c_p \frac{\partial T}{\partial t} - k\,\nabla^2 T = 0$$

$$\frac{\partial T}{\partial t} - D\,\nabla^2 T = 0 \qquad D = \frac{k}{\rho c_p} \quad \left[\frac{L^2}{T}\right] \text{ thermal diffusivity}$$




⇒ Heat equation for ice is non-linear

$$\boxed{\rho c_p(T) \frac{\partial h}{\partial t} - \nabla\cdot\left[k(T)\,\nabla T\right] = 0}$$

The temperature dependence of $c_p$ & $k$ isolates hot plutons.

⇒ keep magma chamber alive longer

see Whittington et al. (2009) Nature, 458(7236)