Hydraulic head

If the density/densities are constant, the equations can be simplified by introduction of the hydraulic head.

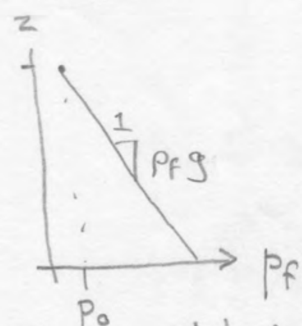
Governing equation for porous flow in an elastic rock:

$$c_r \frac{\partial p_f}{\partial t} - \nabla \cdot \left(\frac{k}{\mu_f} \left(\nabla p_f + \underbrace{\rho_f g \hat{z}}_{\text{density term}} \right) \right) = -\frac{\Delta \rho}{\rho_f \rho_s} \Gamma + c_r \frac{\partial \bar{\sigma}_T}{\partial t}$$

Unknown is p_f, everything else can move to rhs

$$\Rightarrow c_r \frac{\partial p_f}{\partial t} - \nabla \cdot \left(\frac{k}{\mu_f} \nabla p_f \right) = -\frac{\Delta \rho}{\rho_f \rho_s} \Gamma + c_r \frac{\partial \bar{\sigma}_T}{\partial t} + \underbrace{\nabla \cdot \left(\frac{k}{\mu_f} \rho_f g \hat{z} \right)}_{+ \frac{\rho_f g}{\mu_f} \frac{\partial k}{\partial z}}$$

everything except k is constant



Is p_f a flow potential?

Does ∇p_f determine flow direction?

No, because of the density term.

For example: p_f is not constant at hydrostatic equilibrium

Introduce the hydraulic head, which absorbs the density term and is a true flow potential.

Definition of hyd. head: $\boxed{h = z + \frac{p_f - p_0}{\rho_f g}}$ [L]

Substitute into Darcy's law:

$$p_f = p_0 + \rho_f g (h - z) \Rightarrow \nabla p_f = \rho_f g (\nabla h - \underbrace{\nabla z}_{\hat{z}})$$

$$q = -\frac{k}{\mu_f}(\nabla p_f + \rho_f g \hat{z}) = -\frac{k}{\mu_f}(\rho_f g \nabla h \cancel{- \rho_f g \hat{z} + \rho_f g \hat{z}})$$

$$= -\frac{k \rho_f g}{\mu_f} \nabla h = -K \nabla h$$

Darcy's law: $\boxed{q = -K \nabla h}$

hydraulic conductivity: $\boxed{K = \frac{k \rho_f g}{\mu_f}}$ $\quad \frac{\cancel{L^2} \cancel{M} \cancel{L T} L}{\cancel{L^3} \cancel{M} T^{\cancel{2}}} = \frac{L}{T}$

Note: • Permeability, k, is a property of the rock

hyd conductivity, K, is property of rock-fluid system

• h is a velocity potential: • $\nabla h = 0 \Rightarrow q = 0$

• $\nabla h \parallel q$ indicates dir. of flow

Governing equation for head:

$$\boxed{S \frac{\partial h}{\partial t} - \nabla \cdot (K \nabla h) = -\frac{\Delta \rho}{\rho_f \rho_s} \Gamma + c_r \frac{d \bar{\sigma}_T}{dt}}$$

$S = \rho_f g c_r$ specific storage in terms of head $\quad \frac{\cancel{M}}{L^3} \frac{L}{\cancel{T^2}} \frac{L \cancel{T^2}}{\cancel{M}} = \frac{1}{L}$

Overpressure head

For flow in ductile media we have rewritten Darcy's law:

$$q_r = -\frac{k}{\mu_f}(\nabla p + \Delta\rho g \hat{z}) \qquad p = p_f - p_s \qquad \Delta\rho = \rho_f - \rho_s > 0$$

By analogy to the hydr. head we define an

Overpressure head: $\boxed{h = z + \frac{p}{\Delta\rho g}} \Rightarrow p = \Delta\rho g (h - z)$

$$\nabla p = \Delta\rho g (\nabla h - \hat{z})$$

Again we have: $q_r = K \nabla h$ where $K = \frac{k \Delta\rho g}{\mu_f}$

Note the slight differences in definition of h and K!

Substitute into continuity equation:

$$\nabla \cdot (\underline{q}_r + \underline{v}_s) = \nabla \cdot \underline{q}_r + \nabla \cdot \underline{v}_s = -\frac{\Delta\rho}{\rho_f \rho_s}\Gamma$$

$$\nabla \cdot \underline{v}_s = \frac{p}{\xi} = \frac{\Delta\rho g}{\xi}(h - z)$$

so that

$$\boxed{-\nabla \cdot (K \nabla h) + \frac{h}{\Xi} = -\frac{\Delta\rho}{\rho_f \rho_s}\Gamma + \frac{z}{\Xi}} \quad \text{where} \quad \boxed{\Xi = \frac{\xi}{\Delta\rho g}}$$