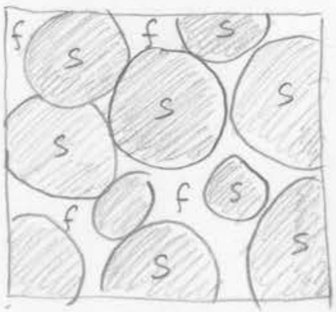
Porous medium Basics



The partially molten ice shell comprises two phases: solid (ice) and fluid (brine).

Porosity / melt fraction:

$$\phi = \frac{V_f}{V_f + V_s} \in [0,1]$$

V_f = fluid volume
V_s = solid volume

$\Rightarrow$ ϕ is fraction of space occupied by fluid
$1-\phi$ is fraction of space occupied by solid

Assume the porous medium is <u>saturated</u>, i.e. entire pore space is filled by fluid.

Assume both phases have constant density, ρ_f and ρ_s, respectively.
$\Rightarrow$ phases are incompressible but two phase mixture is not!

Darcy's law:

$$\boxed{\bar{q}_r = \phi(\bar{v}_f - \bar{v}_s) = -\frac{k}{\mu_f}\left(\nabla p_f + \rho_f g \hat{z}\right)}$$

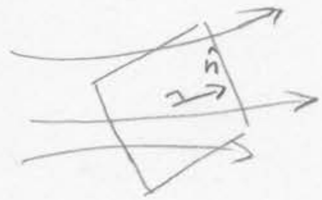
$\bar{q}_r$ = relative volumetric flux of fluid $\left[\frac{L^3}{L^2 T} = \frac{L}{T}\right]$
$\bar{v}_p$ = velocity of phase p $\left[\frac{L}{T}\right]$
p_p = pressure of phase p $\left[\frac{M}{LT^2}\right]$
ρ_p = density of phase p $\left[\frac{M}{L^3}\right]$
g = grav. acceleration $\left[\frac{L}{T^2}\right]$ ~10 on Earth, ~2 on Europa
$\hat{z}$ = unit normal vector in z-dir
k = intrinsic permeability of the rock $[L^2]$
μ_f = dynamic viscosity of fluid $\left[\frac{M}{LT}\right]$

Difference between flux & velocity:

Flow rate: $R = \frac{\text{something}}{\text{Time}}$ (scalar)

"flow rate of your fosset is 1 liter per minute"

Flux: $\bar{q} = \frac{\text{something}}{\text{Area Time}}$ (vector)

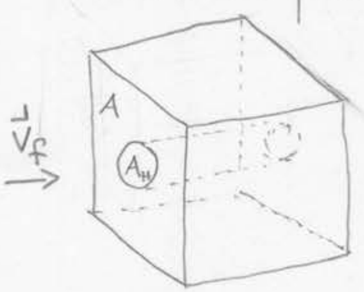


volumetric flux $= \frac{L^3}{L^2 T} = \frac{L}{T}$

has units of velocity but in a porous medium it is different from velocitc

In a rigid porous medium at rest $\bar{v}_s = 0 \Rightarrow \boxed{\bar{q}_r = \bar{q}_f = \phi \bar{v}_f}$

Idealized porous medium: "Block with a hole drilled trough"



Flow rate: $R = A_H \vec{v}_f$ $\left[\frac{L^3}{T}\right]$

Flux: $q = \frac{R}{A} = \frac{A_H}{A} \vec{v}_f = \phi \vec{v}_f$ $\left[\frac{L^3}{L^2 T} = \frac{L}{T}\right]$

Note: Darcy's law is <u>relative</u> to solid, $\Rightarrow \bar{q}_r = \phi(\bar{v}_f - \bar{v}_s)$

In most normal applications $v_s \sim 0$, but in ductile ice $\bar{v}_s$ is not zero.

$\bar{q}_r = \phi(\bar{v}_f - \bar{v}_s)$ = relative fluid flux $\Leftrightarrow$ Darcy's law

$\bar{q}_f = \phi \bar{v}_f$ = absolute fluid flux $\longleftrightarrow$ mass balance