Operators in radial coordinates

Consider PDE for radial flow "near a well.

1) Derivatives explicitly: $-\frac{1}{r}\frac{d}{dr}\left(r\, K(r)\frac{dh}{dr}\right) = f$

2) General div-grad notation: $-\nabla\cdot(K\nabla h) = f$

$\Rightarrow$ operators hide coord. system!

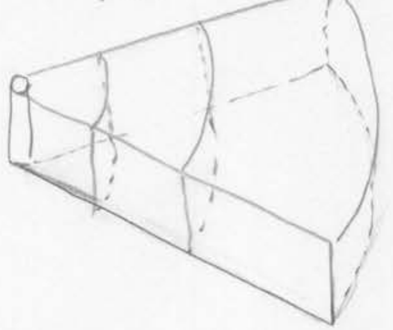
How are they connected?

- gradient: $\nabla() = \frac{d}{dr}$
- divergence: $\nabla\cdot() = \frac{1}{r}\nabla\cdot(r\ )$

Discrete radial divergence:

Understand origin of the two r's:

$$-\frac{d}{dr}\Big(\underset{\underset{\text{faces}}{\uparrow}}{r}\, K\frac{dh}{dr}\Big) = \underset{\underset{\text{volumes}}{\uparrow}}{r} f$$



In radial coordinates both the cell volumes and the areas of the cell faces increase with r.

divergence: $\underset{\underset{\text{outer}}{\uparrow}}{\frac{1}{r}}\nabla\cdot(\underset{\underset{\text{inner}}{\uparrow}}{r}\ )$

inner r: change in face areas
$\Rightarrow$ evaluate at faces

outer r: change in cell volumes
$\Rightarrow$ evaluate at cell centers

$\Rightarrow$ similar to variable coefficients, exept <u>no</u> averaging

Discrete radial divergence: $\underline{\underline{R_0}}\ \underline{\underline{D}}\ \underline{\underline{R_1}}$

1) $\underline{\underline{D}}$: Nx by Nx+1 standard divergence matrix

2) $\underline{\underline{R_1}}$: Nx+1 by Nx+1 diagonal matrix with r at cell faces on diagonal (similar to $\underline{\underline{K_d}}$)

3) $\underline{\underline{R_0}}$ Nx by Nx diagonal matrix with r at cell centers on diagonal

$\Rightarrow$ add this to "build_ops"