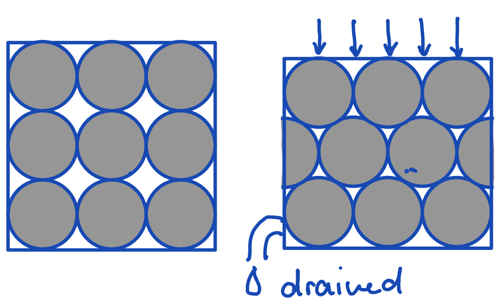
Slightly compressible flow

So far we have considered incompressible flow where both the density of fluid and the porosity do not change with time. Transient behavior arises from compressibility of either rock or fluid.

In general, fluids are more compressible than solids. At pressures of interest both indidual phases are not compressible. Instead compressibility arises from the interaction of the two phases and is termed consolidation.



Consolidation generally involves a change in porosity and is only possible if fluid can be expelled. Consolidation gives rise to an effective compressibility of the porous medium.

Balance of fluid & solid mass

Fluid mass: $\frac{\partial}{\partial t}(\phi \rho_f) + \nabla \cdot [\phi \rho_f \underline{v}_f] = 0$

Solid mass: $\frac{\partial}{\partial t}((1-\phi)\rho_s) + \nabla \cdot [(1-\phi)\rho_s \underline{v}_s] = 0$

Assume $\rho_f = \text{const.}$, $\rho_s = \text{const.}$ and $\underline{q} = \phi(\underline{v}_f - \underline{v}_s)$ (Darcy's law)

Sum both fluid & solid mass balance:

$$\boxed{\nabla \cdot [\phi \underline{v}_f + (1-\phi)\underline{v}_s] = 0}$$ Two-phase continuity eqn

Using the definition of $\underline{q}$ we have:

$$\boxed{\nabla \cdot \underline{q} + \nabla \cdot \underline{v}_s = 0}$$

Flow in an elastic rock

Bulk rock compressibility: $c_r = \frac{1}{V_T} \frac{dV_T}{d\sigma'}\Big|_T \sim 10^{-8} \frac{1}{Pa}$

units of pressure

$\frac{F}{A} = \frac{M \cdot L/T^2}{L^2} = \frac{M}{LT^2}$

$V_T = V_f + V_R$ total rock volume

σ' = effective stress
(stress/weight on solid)

Terzaghi's principle: $\boxed{\sigma_T = \sigma' + p}$

total stress

pore pressure

Volumetric strain rate: $\dot{\varepsilon}_{vol} = \frac{1}{V_T}\frac{dV_T}{dt} = \nabla \cdot \underline{v}_s$

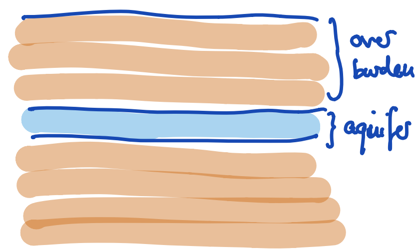
from definition of compressibility: $\frac{1}{V_T} dV_T = -c_r\, d\sigma'$

$$\Rightarrow \quad \nabla \cdot \underline{v}_s = \frac{1}{V_T}\frac{dV_T}{dt} = -c_r \frac{d\sigma'}{dt} = -c_r\left(\frac{d\sigma_T}{dt} - \frac{dp}{dt}\right) = c_r\left(\frac{dp}{dt} - \frac{d\sigma_T}{dt}\right)$$

$$\boxed{\nabla \cdot \underline{v}_s = c_r\left(\frac{dp}{dt} - \frac{d\sigma_T}{dt}\right)}$$

Constitutive equation for elastic rock

Note: σ_T is the total stress/weight on the aquifer due to the over burden.



Convert pressure to head: $h = z + \frac{p - p_0}{\rho g}$

$$\Rightarrow \frac{dp}{dt} = \rho g \frac{dh}{dt}$$

$$\nabla \cdot \underline{v}_s = c_r\left(\rho g \frac{dh}{dt} - \frac{d\sigma_T}{dt}\right)$$

Substitute into continuity eqn together with Darcy's law

$$\nabla \cdot \underline{q} + \nabla \cdot \underline{v}_s = 0$$

$$\rho g\, c_r \frac{dh}{dt} - \nabla \cdot (K \nabla h) = c_r \frac{d\sigma_T}{dt}$$

Specific storage: $S_s = \rho g\, c_r$ $\qquad \frac{\cancel{M}}{L^{\cancel{3}}}\,\frac{\cancel{L}}{\cancel{T^2}}\,\frac{\cancel{L}\cancel{T^2}}{\cancel{M}} = \frac{1}{L}$

Physical interpretation:

S_s is the volume of fluid released / stored per unit volume of rock per unit decrease/increase in head.

$c_r \sim 10^{-8} \frac{m s^2}{kg}$ $\quad \rho \sim 10^3 \frac{kg}{m^3}$ $\quad g \sim 1 \frac{m}{s^2}$

$\Rightarrow \quad S_s \approx 10^{-8+3+1} \frac{1}{m} \doteq 10^{-4} \frac{1}{m}$

For 1 m drop in head ~ 100 ml of water are released from the rock due to consolidation.

Note: Here we assume consolidation is reversible!

This means increasing the head will store the same amount of water.

(In real life this is a major problem.)

Transient slightly compressible flow equation

$$\boxed{S_s \frac{dh}{dt} - \nabla \cdot (K \nabla h) = c_r \frac{d\sigma_T}{dt}}$$

Typically the overburden does not change with time $\frac{d\sigma_T'}{dt} = 0$. But there is a potentially interesting application to impact formation.