Solving full system of governing eqns

Dimension less system of equations:

1) $\frac{\partial \phi_D}{\partial t_D} + Pe\, \nabla_D \cdot (\phi_D \underline{v}_D) = Da\, \Gamma_D + \phi_D^m (h_D - z_D)$ } Transport problem transient, local

2) $-\nabla_D \cdot (\phi_D^n \nabla_D h_e) + \phi_D^m h_D = -\Gamma_D + \phi_D^m z_D$

3) $-\nabla_D^2 u_D = \phi_D^m (h_D - z_D)$ } Flow problem instantaneous, global

where $\underline{v}_D = -\nabla_D u_D$ and $q_D = -\phi_D^n \nabla h_D$ $\quad p_D - h_D\ z_D$

Domain $z_D \in [0, \mathcal{Z}]$ $\quad t_D \in [0, \mathcal{T}]$

Dimensionless parameters: $Pe = \phi_c$ $\quad Da = \frac{p_t}{\Delta p}$ $\quad \mathcal{Z} = \frac{H}{\delta}$

Constitutive laws: $K_D = \phi_D^n$ $\quad \Xi_D = \frac{1}{\phi^m}$

Boundary conditions for a compacting column:

No flow at top & bottom: $(\nabla h_D \cdot \hat{n}_z|_{z_D = 0, \mathcal{Z}} = 0$

$(\nabla u_D \cdot \hat{n}_z|_{z_D = \mathcal{Z}} = 0$ $\quad u_D(z_D = 0) = 0$ eliminates constant

no BC's on ϕ_D because $v_D(z_D = 0, \mathcal{Z}) = 0$

Initial condition: $\phi_D(z_D, 0) = 1$

For compacting column: $Da = 0$, $\Gamma_D = 0$

Numerical solution strategy

1) Solve Flow probem

- given a porosity field: ϕ_D

1a) solve mod Helmholtz eqn $\rightarrow h_D$ & p_D

1b) solve Poisson eqn $\rightarrow u_D$ & v_D

2) Solve Transport problem

update the porosity ϕ_D

later we will add transport problem for oxidant concentration

Discretizing Advection eqn with source term

$$\frac{\partial \phi_D}{\partial t_D} + Pe \nabla_D \cdot (\underline{v}_D \phi_D) = \phi_D^m p_D \qquad \underline{\underline{m=1}} \Rightarrow \text{linear eqn}$$

$$\underline{\underline{I}} \frac{\phi_D^{k+1} - \phi_D^k}{\Delta t} + Pe\ \underline{\underline{D}} * \underline{\underline{A}}(\underline{v}) [\theta \phi^k + (1-\theta)\phi^{k+1}] = \underline{\underline{P}} [\theta \phi^k + (1-\theta)\phi^{k+1}]$$

where $\underline{\underline{P}}$ = spdiags(pD, 0, N, N)

separate k+1 and k terms

$$\underbrace{\left\langle \underline{\underline{I}} + \Delta t (1-\theta) [\underbrace{Pe\ \underline{\underline{D}} * \underline{\underline{A}}(\underline{v}) - \underline{\underline{P}}}_{\underline{\underline{L}}}] \right\rangle}_{\underline{\underline{IM}}} \phi^{k+1} = \underbrace{\left\langle \underline{\underline{I}} - \Delta t \theta [\underbrace{Pe\ \underline{\underline{D}} * \underline{\underline{A}}(\underline{v}) - \underline{\underline{P}}}_{\underline{\underline{L}}}] \right\rangle}_{\underline{\underline{EX}}} \underline{\phi}^k$$