Variable coefficients

- Heterogeneity, ie variation of physical properties with location, is a key element of flow in porous media.
- In ductile media the evolution of the porosity will naturally lead to heterogeneity

Dimensionless governing equations:

1) $\frac{\partial \phi_D}{\partial t} + Pe \, \nabla \cdot (\phi_D \underline{v}_D) = Da \, \Gamma_D + \phi^m (h_D - z_D)$

2) $-\nabla_D \cdot (\phi_D^n \nabla_D h_D) + \phi_D^m h_D = -\Gamma_D + \phi_D^m z_D$

3) $-\nabla_D^2 u_D = \phi_D^m (h_D - z_D)$

where $\underline{v}_D = -\nabla_D u_D \qquad q_D = -\phi_D^m \nabla_D h_D$

To cases: 1) ϕ_D multiplies unknown: $\Xi_D = \frac{1}{\phi_D^m}$

2) ϕ_D multiplies gradient of unknown: $K_D = \phi_D^n$

Case 1: ϕ_D and h_D are both located in cell center

Example $\phi_D^m h_D$ in mod Helmholtz eqn

Just a element wise multiplication $\underline{\phi}_D^m .* \underline{h}_D$,

but to form $\underline{\underline{L}}$ we need to write it as a matrix vector product

$\underline{\phi}_D^m .* \underline{h}_D = \underline{\underline{\text{Phi\_m}}} * \underline{h}_D$

$$\underline{\underline{\text{Phi\_n}}} = \begin{pmatrix} \ddots & & \\ & \underline{\phi}_D^m & \\ & & \ddots \end{pmatrix} \quad N \text{ by } N$$

Case 2: ϕ_D is known in cell center and needs to be averaged to cell faces where the gradient is evaluated!

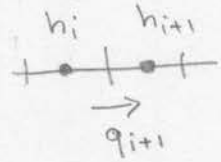
Example: $-\nabla_D \cdot \phi_D^n \nabla_D h_D$ term in mod. Helmholtz eqn

Discrete analog: $-\underset{N_x \cdot N_{fx}}{\underline{D}} * \underset{N_{fx} \cdot N_{fx}}{\underline{\underline{\text{Phi\_n}}}} * \underset{N_{fx}\ N_x}{\underline{\underline{G}}} * \underset{N_x\ 1}{\underline{h_D}}$

$$\text{Phi\_n} = \begin{pmatrix} \ddots & & \\ & \langle \phi^n \rangle & \\ & & \ddots \end{pmatrix}$$

Diagonal Nf by Nf matrix with the appropriate average of ϕ^n on the diagonal.

What is the appropriate average?



$$q_{i+1} = -(\phi^n)_{i+\frac{1}{2}} \frac{h_{i+1} - h_i}{\Delta x}$$

arithmetic average: $(\phi^n)_{i+\frac{1}{2}} = \dfrac{\phi_i^n + \phi_{i+1}^n}{2}$

harmonic average: $(\phi^n)_{i+\frac{1}{2}} = \dfrac{2}{\frac{1}{\phi_i^n} + \frac{1}{\phi_{i+1}^n}}$

In porous media we choose the harmonic average because it preserves thin flow barriers.

This is because harmonic average is biased to the lowest porosity.

$\Rightarrow$ think electrical resistors in series