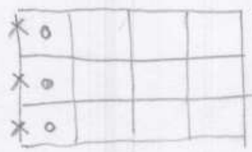
Assembly of $\underline{\underline{Ax}}^+$, $\underline{\underline{Ax}}^-$, $\underline{\underline{Ay}}^+$ and $\underline{\underline{Ay}}^-$ with Kronecker products

Ax - matrices

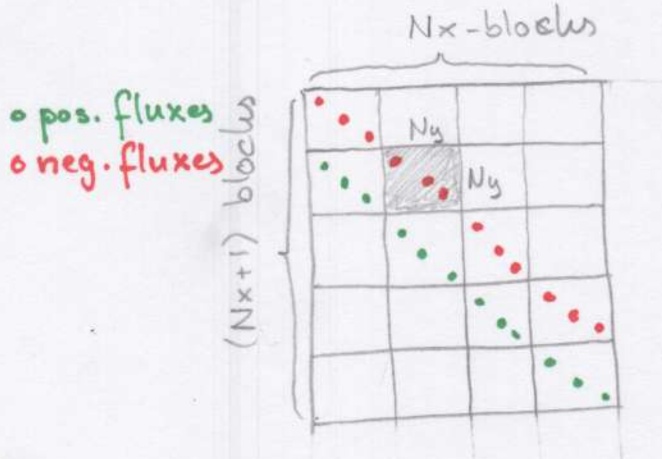
Ax computes Ny by (Nx+1) fluxes from Ny by Nx concentration

Grid



Nx columns of Ny concentration

Nx+1 colums of Ny fluxes



Each block is Ny by Ny

$\underline{\underline{I_y}}$ = speye(Ny);

Positive fluxes are on -1 block diagonal

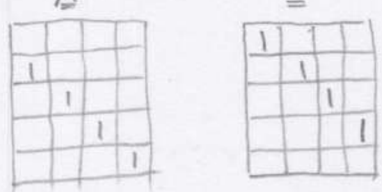
1D $\underline{\underline{A}}^+$ matrix:

Axp1 = spdiags(ones(Nx,1), -1, Nx+1, Nx);

1D $\underline{\underline{A}}^-$ matrix:

Axn1 = spdiags(ones(Nx,1), 0, Nx+1, Nx);

1D matrices: $\underline{\underline{A}}^+$ $\underline{\underline{A}}^-$

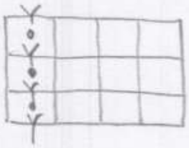


2D matrices: Axp = kron(Axp1, Iy);

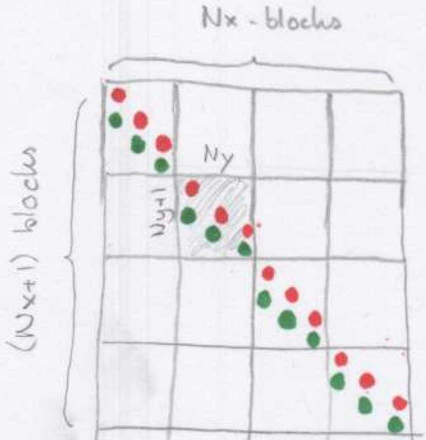
Axn = kron(Axn1, Iy);

Ay-matrices

Ay computes Nx columns of Ny+1 fluxes from
Nx columns of Ny concentrations



$\Rightarrow \underline{\underline{A_y}}$ is Nx by Nx block matrix
with blocks of size Ny+1 by Ny



Overall block structure

$\underline{\underline{I_x}}$ = speye(Nx);

Each block is 1D Matrix:

Axp1 = spdiags(ones(Ny,1),-1, Ny+1, Ny);

Axn1 = spdiags(ones(Ny,1), 0, Ny+1, Ny);

o neg. fluxes o pos. fluxes

Assemble 2D y-matrices: Ayp = kron(Ix, Ayp1);

Ayn = kron(Ix, Ayn1);

Assemble overall 2D matrices:

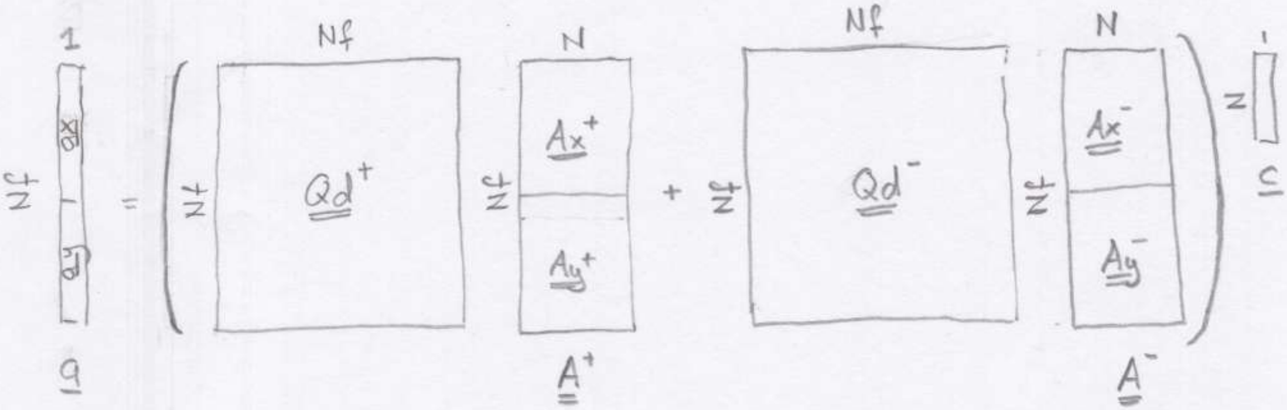
$\underline{\underline{A_p}} = \begin{bmatrix} \underline{\underline{A_{xp}}} \\ \underline{\underline{A_{yp}}} \end{bmatrix} \qquad \underline{\underline{A_n}} = \begin{bmatrix} \underline{\underline{A_{xn}}} \\ \underline{\underline{A_{yn}}} \end{bmatrix}$

Kronecker product assembly of advection matrices

Problem: In $\underline{\underline{D}}$ and $\underline{\underline{G}}$ the matrix blocks are identical, but in $\underline{\underline{A}}$ each block has same structure but values differ because q's vary across the domain.

Solution: Separate the structure, i.e. the 1's and 0's from the magnitudes.

The overall scheme for computing advective fluxes $\underline{a}$:



Where we have following sparse matrices:

$\underline{\underline{Qd}}^+$ = Nf by Nf matrix with positive fluxes on diagonal
$\underline{\underline{Qd}}^-$ = Nf by Nf matrix with negative fluxes on diagonal } magnitudes
$\underline{\underline{A}}^+$ = Nf by N matrix with ones in location of pos. fluxes
$\underline{\underline{A}}^-$ = Nf by N matrix with ones in location of neg. fluxes } structure

If flow is evolving only $\underline{\underline{Qd}}^+$ and $\underline{\underline{Qd}}^-$ must be updated!

```
Qdp = spdiags(max(q,0), 0, Nf, Nf);
Qdn = spdiags(min(q,0), 0, Nf, Nf);
```

So that : $\boxed{\underline{\underline{A}}(q) = \underline{\underline{Qdp}}(q) * \underline{\underline{Ap}} + \underline{\underline{Qdn}}(q) * \underline{\underline{An}}}$