Thermal boundary layer beneath Mid-Ocean Ridge

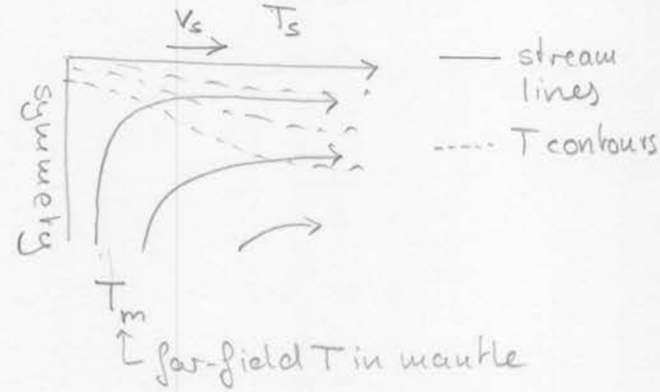
Idea is to add an actual corner flow velocity distribution to the models of the thermal structure of the oceanic plates.

Governig equations:

PDE's: 1) $\nabla \cdot [\mu (\nabla \underline{v} + \nabla^T \underline{v})] - \nabla p = 0$

2) $\nabla \cdot \underline{v} = 0$

3) $\nabla \cdot (\underline{v}T - k \nabla T) = 0$



Problem: There is no obvious external length scale in this problem.
We just want the domain to be large enough that the other boundaries do not affect solution near the corner.

Solution: Look for an internal length scale
$\Rightarrow$ thickness of thermal boundary layer.

Dimension Less variables: $\underline{v}' = \frac{\underline{v}}{V_s}$ $\quad p' = \frac{p}{p_c}$ $\quad x' = \frac{x}{x_c}$ $\quad T' = \frac{T - T_s}{\Delta T}$ $\quad \Delta T = T_M - T_s$

1) $\frac{\mu V_s}{x_c^2} \nabla' \cdot [\nabla' \underline{v}' + \nabla'^T \underline{v}'] - \frac{p_c}{x_c} \nabla' p' = 0$

$$\nabla' \cdot [\nabla' \underline{v}' + \nabla'^T \underline{v}'] - \frac{p_c x_c}{\mu V_s} \nabla' p' = 0 \quad \Rightarrow \quad \boxed{p_c = \frac{\mu V_s}{x_c}}$$

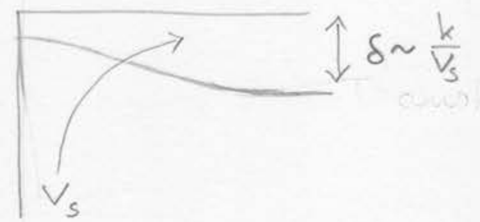
$$\boxed{\nabla' \cdot [\nabla' \underline{v}' + \nabla'^T \underline{v}'] - \nabla' p' = 0}$$

2) $\frac{V_c}{x_c} \nabla' \cdot \underline{v}' = 0 \quad \rightarrow \quad \boxed{\nabla' \cdot \underline{v}' = 0}$

3) $\nabla' \cdot \left[\frac{v_s \cancel{\Delta T}}{x_c} \underline{v}' T' - \frac{k \cancel{\Delta T}}{x_c^2} \nabla' T' \right] = 0$

$$\nabla' \cdot \left[\underline{v}' T' - \frac{k}{v_s x_c} \nabla' T' \right] = 0 \quad \Rightarrow \quad \boxed{x_c = \frac{k}{v_s}}$$

This is an internal length scale for the thickness of the thermal boundary layer, $\delta \sim \frac{k}{v_s}$



It increases with thermal diffusivity, k, and deacreases with increasing plate velocity, v_s.

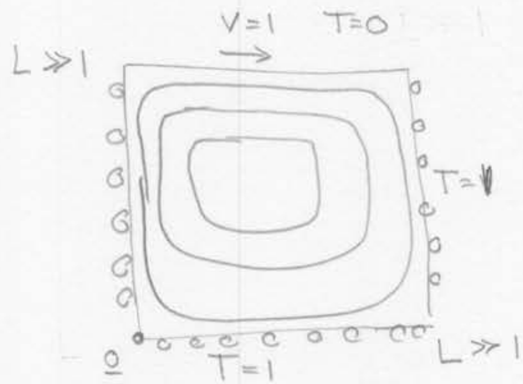
Dimensionless problem

after dropping the primes

1) $\nabla \cdot [\nabla \underline{v} + \nabla^T \underline{v}] - \nabla p = 0$

2) $\nabla \cdot \underline{v} = 0$

3) $\nabla \cdot [\underline{v} T - \nabla T] = 0$



Driven by unit velocity at top and slip bc's elsewhere.

We need to determine L by trial and error, i.e. choose it large enough that solution near top left corner becomes independent of L. Start with $L = 100$.