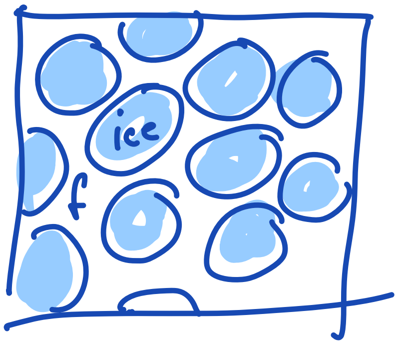
Lecture 1: Intro to porous media



Partially molten ice

comprises 2 phases:

solid (ice) and fluid (brine)

water

Volume fractions: $\phi_f = \dfrac{V_f}{V_f + V_s} \in [0,1]$

extpect $\phi_f \sim 0.01 - 0.1$

$\phi_f = \phi$ porosity

$\phi_s = \dfrac{V_s}{V_f + V_s} \in [0,1]$

Volume fraction constraint: $\phi_f + \phi_s = 1$

If the fluid fills entire pore space

$\Rightarrow$ saturated porous medium

Assume both phases are incompressible

$\rho_f \neq \rho_f(p)$ $\quad\quad$ $\rho_s \neq \rho_s(p)$ $\quad\quad$ p = pressure

$\Rightarrow$ although phases are incompressible the two phase mixture is not ∇!

Darcy's law: $\boxed{\underline{q}_r = \phi(\underline{v}_f - \underline{v}_s) = -\frac{k}{\mu_f}(\nabla p_f + \rho_f g \hat{z})}$

$\underline{q}_r$ = rel. volumetric fluid flux $\left[\frac{L^3}{L^2 T} = \frac{L}{T}\right]$

$\underline{v}_p$ = velocity of phase p $\left[\frac{L}{T}\right]$

p_p = pressure of phase p $\left[\frac{M}{L T^2}\right]$

ρ_p = density of phase p $\left[\frac{M}{L^3}\right]$

g = grav. acceleration $\left[\frac{L}{T^2}\right]$ ~10 Earth, ~2 Europa

$\left(\nabla z \quad \frac{L}{L}\right)$ $\hat{z}$ = unit vector in z-dir $[1]$

k = intrinsic permeability $[L^2]$

μ_f = dyn. viscosity of fluid $\left[\frac{M}{LT}\right]$

→ gradient flow similar to Fourier's law

or Ohm's law

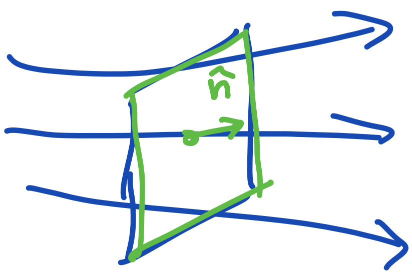
Difference between flux & velocity

Flow rate: $R = \frac{\text{something}}{\text{time}}$ scaler $\left[\frac{\#}{T} = \frac{L^3}{T}\right]$

"spring has a flow rate of $15\ m^3/s$

hydrology rate = discharge Q

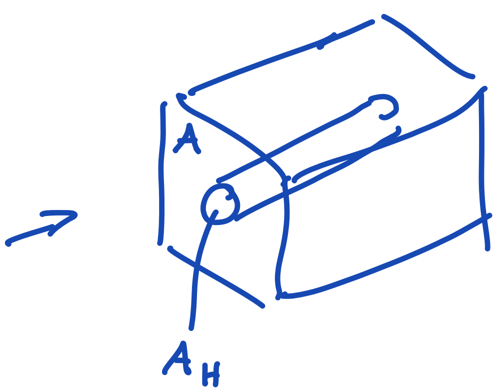
Flux: $q = \frac{\text{something}}{\text{Area Time}} = \frac{Q}{A}$ $\left[\frac{\#}{L^2 T} \rightarrow \frac{L^3}{L^2 T} = \frac{L}{T}\right]$



Flux has units of velocity but nothing moves with that velocity!

In a rigit porous medium at rest

$$q_r = q_f = \phi\, v_f \qquad \rightarrow \qquad v_f = \frac{q_f}{\phi}$$



Flow rate: $R = v_f A_H$ $\quad \frac{L}{T} L^2$

Flux: $q_f = \frac{R}{A} = \underbrace{\frac{A_H}{A}}_{\phi} v_f$

Think of water hose