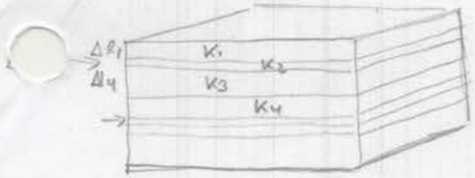
Simplest Layered Medium



Stack of N layers with thickness $\Delta\ell_i$ and conductivity K_i, $[i = 1, 2, \dots N]$

How does this affect flow?

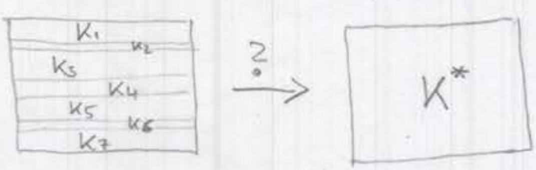
Generally 3D problem $\Rightarrow$ computation

We can look at 2 limiting cases:

1) Flow perpendicular to the layers (in-series)
2) Flow parallel to layers (in-parallel)

} flow is 1D

To understand the effect of the layering we try to find an effective property that represents the entire stack of layers?

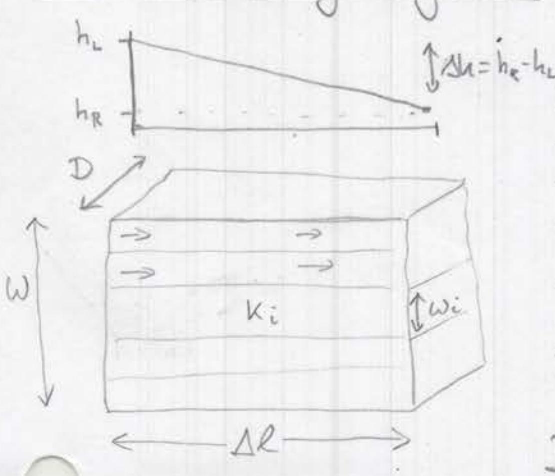


Would K^* be the same for flow parallel to layers and for flow perpendicular to layers?

Fine scale: layered medium K changes with location (heterogeneous)

Coarse scale: K^* changes with the direction of flow (anisotropic).

1) Flow along layers



Apply a const. head difference horizontally across the sample. $\Delta h = h_R - h_L$. Each layer has same $h(x)$.

Top, bottom, front & back are closed; i.e. no flow.

$\Rightarrow$ 1D flow along each layer

$\Rightarrow$ consider them separately

Darcy in i-th layer: $Q_i = -D w_i K_i \frac{\Delta h}{\Delta L}$

Darcy for whole stack: $Q = -DW K^*_{\parallel} \frac{\Delta h}{\Delta L}$

$K^*_{\parallel}$ = effective conductivity for flow along layers

Total flow : ① $Q = \sum_{i=1}^{N} Q_i = -\sum_{i=1}^{N} D w_i K_i \frac{\Delta h}{\Delta L} = -D \frac{\Delta h}{\Delta l} \sum_{i=1}^{N} w_i K_i$

② $Q = -D W K_{\parallel}^{*} \frac{\Delta h}{\Delta L}$

equate ① & ② $-\cancel{D} \frac{\Delta h}{\Delta L} \sum_{i=1}^{N} w_i K_i = -\cancel{D} W K_{\parallel}^{*} \frac{\Delta h}{\Delta L}$

$$\sum_{i=1}^{N} w_i K_i = W K_{\parallel}^{*}$$

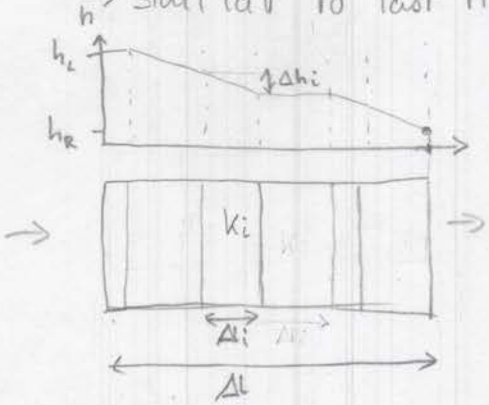
$$\Rightarrow \boxed{K_{\parallel}^{*} = \sum_{i=1}^{N} \frac{w_i}{W} K_i}$$

Effective hydraulic conductivity for flow along layers is an arithmetic average of layer K_i's weighted by the fractional width of the layer $\frac{w_i}{W}$.

⇒ high K layers dominate the behavior

2) Flow across layers

$\vec{n}$ → similar to last time



Apply uniform horizontal head difference $\Delta h = h_R - h_L$ and no flow across other sides
⇒ 1D flow perpendicular to the layers.

Area is constant ⇒ q is const. in each layer

Darcy in ith layer: $q = -K_i \frac{\Delta h_i}{\Delta l_i}$

Darcy across whole stack: $q = -K_{\perp}^{*} \frac{\Delta h}{\Delta L}$

$K_{\perp}^{*}$ = effective conductivity for flow across layers

$$\Delta h = \sum_{i=1}^{N} \Delta h_i \qquad \Delta h_i = -q \frac{\Delta l_i}{K_i}$$

$$K_{\perp}^{*} = -q \frac{\Delta L}{\Delta h} = -q \frac{\Delta L}{\sum_{i=1}^{N} \Delta h_i} = -q \frac{\Delta L}{\sum_{i=1}^{N} -q \frac{\Delta l_i}{K_i}} = \frac{\Delta L}{\sum_{i=1}^{N} \frac{\Delta l_i}{K_i}} = \frac{1}{\sum_{i=1}^{N} \frac{\Delta l_i / \Delta l}{K_i}}$$

$$\boxed{K_{\perp}^{*} = \frac{1}{\sum_{i=1}^{N} \frac{\Delta l_i / \Delta L}{K_i}}}$$

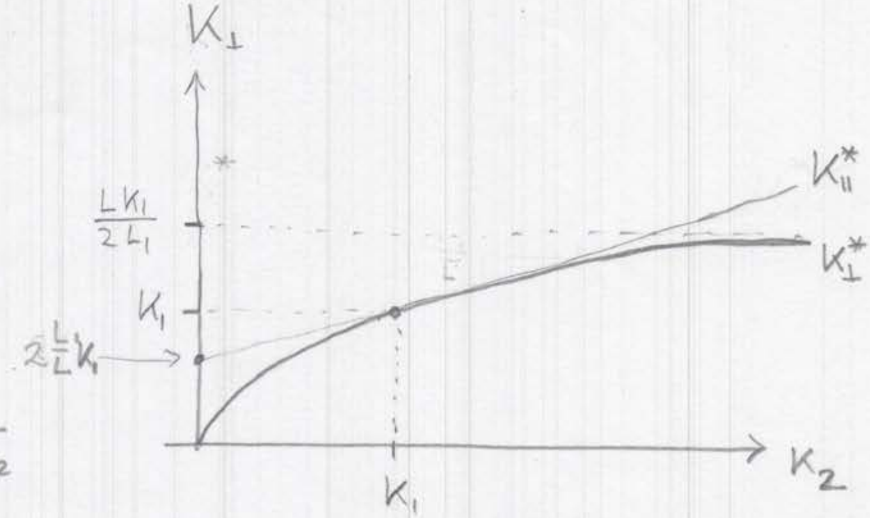
Effective hydraulic conductivity for flow across layers is a harmonic average of the K_i's of the layers weighted by the fractional width of layer.
⇒ lowest K will dominate

Compare $K_{\parallel}^*$ and $K_{\perp}^*$

Consider 3 layer medium

L_1	L_2	L_1
K_1	K_2	K_1

$L = 2L_1 + L_2$



$$K_{\parallel}^* = \frac{L_1}{L}K_1 + \frac{L_2}{L}K_2 + \frac{L_1}{L}K_1$$
$$= 2\frac{L_1}{L}K_1 + \frac{L_2}{L}K_2$$

$$K_{\perp}^* = \frac{L}{\frac{2L_1}{K_1} + \frac{L_2}{K_2}} = \frac{LK_2}{2L_1\frac{K_2}{K_1} + L_2}$$

limit $K_2 \to 0$: $K_H^* = 2\frac{L_1}{L}K_1$

$K_{\parallel}^* = 0$

limit $K_2 \to \infty$: $K_{\parallel}^* \to \infty$

$$K_{\perp}^* = \frac{L}{\frac{2L_1}{K_1} + \frac{L_2}{K_2}\nearrow 0} = \frac{LK_1}{2L_1}$$

Physical Intuition:

Flow across layers: single low-k layer can block flow

Flow along layers: single high-K layer can lead to a lot of flow

$\Rightarrow$ shale layers and fractures are so important!