Gravity

In many planetary problems $\underline{g}$ varies and has to be computed by solving the original Poisson equation for the gravitational field:

Gravitational field: $\underline{g} = -\nabla \Phi$ (vector field)

where Φ is gravitational potential

Gauss' law of gravity

$$\nabla \cdot \underline{g} = -4\pi G \rho$$

where G is gravitational constant

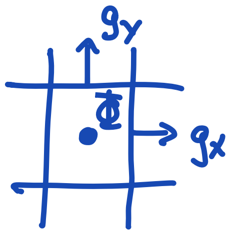
Combine to obtain Poisson's eqn for gravity

$$\boxed{\nabla^2 \Phi = 4\pi G \rho}$$

Similar to groundwater:

$$\underline{g} \rightarrow \underline{q}$$
$$\Phi \rightarrow h$$

If we discretize



$g = \begin{bmatrix} g_x \\ g_y \end{bmatrix}$

$$\underline{\underline{D}} * \underline{\underline{G}} * \underline{\Phi} = 4\pi G \rho$$

$$\underline{\underline{L}}\, \underline{\Phi} = \underline{f_s}$$

$$g = -\underline{\underline{G}} * \underline{\Phi}$$

$\Rightarrow$ g is like a flux and lives on faces

Gravity term in Darcy's law

$$q = -\frac{k}{\mu}\left(\nabla p + \rho_f g \,\hat{z}\right)$$

scalar $\quad g = |g|$

Reformulate Darcy's law interms of g

$$q = -\frac{k}{\mu_f}\left(\nabla p + \rho_f g \,\hat{z}\right)$$

scaler $\quad$ vector $\quad \hat{z} = \nabla z$

introduce: $\boxed{g = -g \nabla z}$

vector $\quad$ scaler

Note: <u>Minus sign</u> because g points downwards

Darcy's law in terms of gravitational field vector

$$\boxed{q = -\frac{k}{\mu_f}\left(\nabla p - \rho_f g\right)}$$

substituting into the mass balance

$$\nabla \cdot q = f_s$$

we have the following Poisson eqn. for pressure

$$\boxed{-\nabla \cdot \left[\frac{k}{\mu_f}\nabla p\right] = f_s - \nabla \cdot \left[\frac{k}{\mu_f}\rho_f g\right]}$$

k = intrinsic permeability $\rightarrow x_c$

μ_f = dynamic viscosity of fluid

here we assume it is constant, but generally

function of T $\rightarrow x_c$

ρ_f = density of fluid, depends on T $\rightarrow x_c$

g = gravitational field vector $\rightarrow x_f$

$$\lambda = \frac{k}{\mu_f} = \text{mobility} \rightarrow x_c$$

Discretization of lhs is the same

$$-\nabla\cdot[\lambda \nabla p] \approx -\underline{\underline{D}} * \underset{\substack{\uparrow \\ \text{harm. average of } \lambda\text{'s}}}{\underline{\underline{Kd}}} * \underline{\underline{G}} * \underline{p} = \underline{\underline{L}} * \underline{p}$$

Discretize rhs:

$$f_s \underbrace{- \nabla\cdot\left[\frac{k}{\mu_f}\rho_f g\right]}_{f_g} \approx \underline{f}_s + \underline{f}_g$$

$\Rightarrow$ new rhs. vector $\underline{f}_g$ due to gravity

New variables: grav = g
grav_vec = $\underline{g}$ $\Big\}$ $g = |\underline{g}|$

$$\boxed{\underline{f}_g = -\underline{\underline{D}} * \underline{\underline{Kd}} * \underline{\underline{Rho}} * \underline{\text{grav\_vec}}}$$

$\underline{\text{grav\_vec}} = \underline{\underline{G}} *$ Grid.xc in 1D
$= \underline{\underline{G}} *$ Yc(:) in 2D
where Yc is generated by meshgrid

$\underline{\underline{\text{Lam}}}$ = comp_mean($\underline{\text{Lam}}$, $\underline{\underline{M}}$, -1 , Grid, 1);
$\underline{\underline{\text{Rho}}}$ = comp_mean($\underline{\text{rho}}$, $\underline{\underline{M}}$, 1 , Grid, 1);

Note: • It is not entirely obvious how $\underline{f}$ on faces should be estimated

• f_g is zero on boundaries