Lecture 1: Porous Media & Darcy's Law

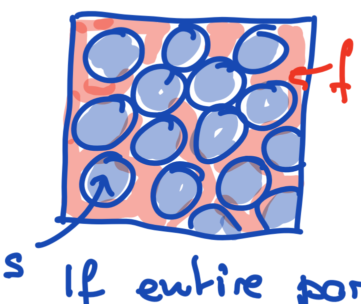
Logistics: - set office hrs
- make sure you are on Piazza & Matlab Grader

Today: - Introduction to porous media
- Darcy's law
- ~~Conservation laws~~

Intro to porous media

⇒ everything we talk about is in ground

Porous media



→ solid matrix (sand grains)

→ pore space (holes in between)

If entire pore space is filled with pore fluid

⇒ saturated porous medium

Note: We consider large sales

an do not model flow on pore scale (μm)

We zoom out to Darcy scale

Volume fractions: $\phi_p = \frac{V_p}{V_T}$

V_p = volume of phase

$V_T = \sum_p V_p$

$\Rightarrow \sum_p \phi_p = 1$ vol. fraction constraint

$\phi_f + \phi_s = 1$

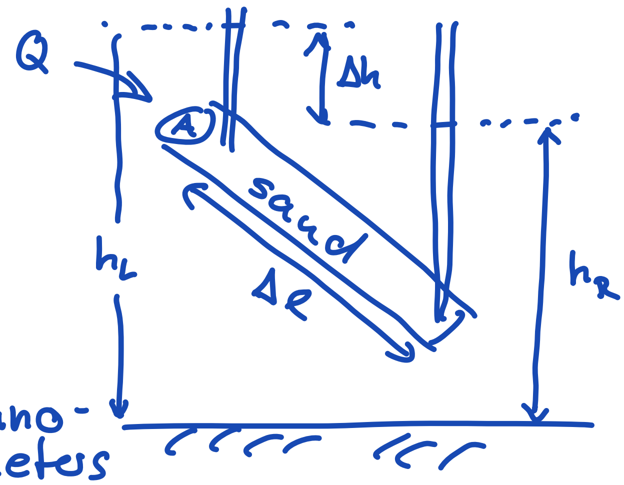
Porosity: $\phi = \phi_f$ (saturated medium)

↑ vol. of pore fluid

Darcy's law

Empirical relation for pore fluid flow on scales much larger than pore scale.

$\Rightarrow$ "Darcy scale"



Δl = sample length [L]

A = sample x section [L^2]

h_L, h_R = water elevations in left & right manometers

⇒ hydraulic heads [L]

$\Delta h = h_R - h_L$ head change [L]

Q = discharge / volumetric flow rate $[\frac{L^3}{T}]$

Experimental observations:

1) $Q \sim -\Delta h$

2) $Q \sim \frac{1}{\Delta L}$

3) $Q \sim A$

$$Q \sim -A \frac{\Delta h}{\Delta L}$$

$$\frac{L^3}{T} \qquad L^2 \; \frac{L}{L}$$

need to introduce constant of proportionality

K $[\frac{L}{T}]$ hydraulic conductivity

Darcy's law:

$$\boxed{Q = -KA\frac{\Delta h}{\Delta L}}$$

Comments:
1) Empirical law (o.k.)
2) Macroscopic law (good)
3) Q is an integrated quantity
⇒ depends on A

For continuum theories we need fluxes not rates.

Rate: amount of something per time

$\frac{\#}{T}$ $\quad Q \quad \frac{L^3}{T}$ scalar

Flux: amount of something per time per area

$\frac{\#}{L^2 T}$ $\rightarrow$ vector

example: specific discharg

$$q = \frac{Q}{A} \hat{n}_A \qquad \frac{L^3}{L^2 T} = \frac{L}{T}$$

volumetric flux

Note: q is not the flow velocity

$$\underline{v} = \frac{q}{\phi}$$