Lecture 14: 2D cylindrical coordinates

Logistics: - HW 6 due Th (2/11)

$\rightarrow$ any issues?

Last time: Stream function

$$\psi(\underline{x}) = \psi_0(\underline{x}_0) + \boxed{\int_\Gamma \underline{q}\cdot\hat{n}\, ds}$$

ψ is path independent if $\nabla\cdot\underline{q} = 0$

$$\Rightarrow \boxed{\psi(\underline{x}) = \psi_0(\underline{x}_0) - \int_{x_0}^{x} q_y\, dx + \int_{x_0}^{x} q_x\, dy}$$

FTC: $\Rightarrow$ $\boxed{\dfrac{\partial\psi}{\partial x} = -q_y \qquad \dfrac{\partial\psi}{\partial y} = q_x}$

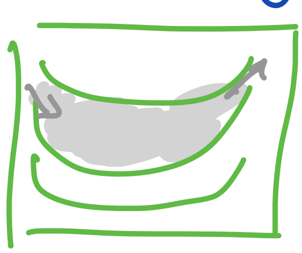
Properties: 1) $|\nabla\psi| = |\underline{q}|$

2) $\boxed{\nabla\psi \perp \underline{q}}$

$\Rightarrow$ allows "equally" spaced stream lines

Today: - Computation of streamfunction

- 2D cylindrical discrete operators

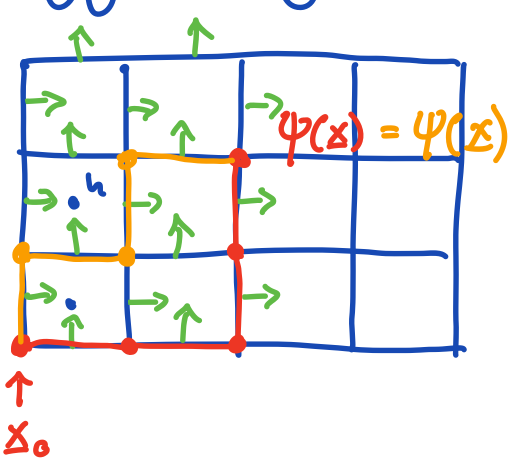


stream tubes = area between two streamlines

Computing the streamfunction

Note: compute ψ as a post-processing step
solely for visualization.

Staggered grid



$\psi(\underline{x}_0) = \psi_0 = \psi_0$

$$\frac{\partial\psi}{\partial x} \Rightarrow v_y \qquad \frac{\partial\psi}{\partial y} \Rightarrow v_x$$

$$\psi(\underline{x}) = \psi_0(\underline{x}_0) - \int_{x_0}^{x} q_y \, dx + \int_{y_0}^{y} q_x \, dy$$

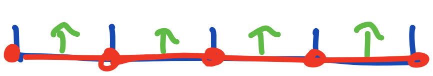
$\Rightarrow$ integrate fluxes along cell faces to avoid interpolation.

$$\underline{v} = \frac{q}{\phi}$$

$$q = \phi \, \underline{v}$$

average velocity

Don't need numerical integrators
Simple Riemann sum is o.k.



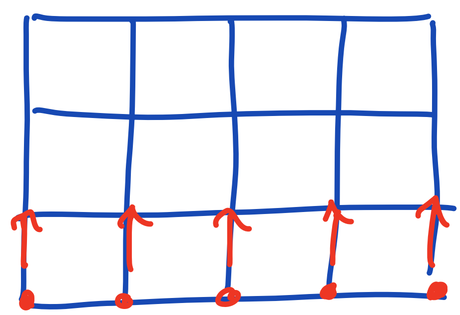
dof_f_ymin

Qymin = Grid.A(dof_f_ymin) .* q(dof_f_ymin)

$\psi_{ymin} = \psi_0 + \mathrm{cumsum}(Q_{ymin})$

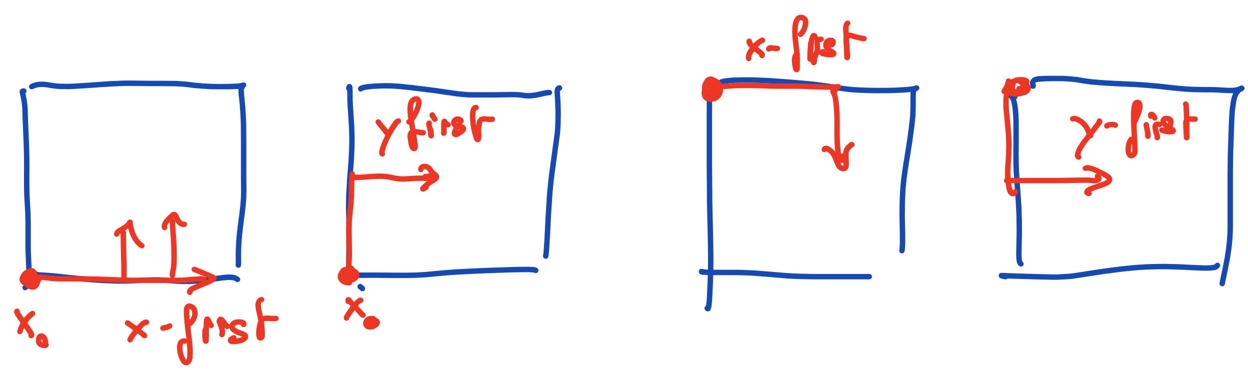
$a = [1, 2, 3, 4]$

$\text{cumsum}(a) = [1 \quad 3 \quad 6 \quad 10]$



another cumsum for each column of y-faces. This can be done in one sweep by formatting the y-fluxes into a matrix $\underline{\underline{Qx}}$

⇒ fast and does not require the solution of a linear system. (post processing step)



Two additional inputs into Grid.

Grid.psi_x0.

Grid.psi_dir (coord. direction)