Lecture 15 : Energy Conservation

Logistics: HW6 is due (10/11)

HW7 will be posted

Last time: - Stream function

- numerical computation
- Examples of flow nets
- path independence

$\Rightarrow$ Flow problem $\rightarrow$ $\underline{v}$ or $\underline{q}$

Today: New Topic $\rightarrow$ new equation

$\Rightarrow$ Energy conservation

heat transport

Energy Conservation Equation in a Porous Medium

Internal Energy: Energy of a body <u>not</u> associated with kinetic or potential energy.

Internal energy → thermal energy / heat

symbol: U units: Joule $\left[\frac{ML^2}{T^2}\right]$

specific internal energy / energy density

$u = \frac{U}{m}$ m = mass $\frac{J}{Kg}$ $\left[\frac{L^2}{T^2}\right]$

Under some assumptions:

$$\boxed{du = c_p \, dT}$$

T = temperature

c_p = specific heat capacity at const. p $\left[\frac{J}{Kg\,K} = \frac{L^2}{T^2 \Theta}\right]$

Physical interpretation:

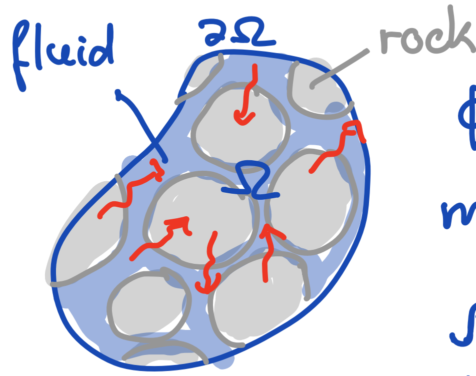
c_p is the heat required to raise the temperature of 1 kg of material by 1 degree K.

Energy density: $\boxed{u(T) = u_0 + c_p (T - T_0)}$

c_p = cost. u_0 = ref. energy T_0 = ref. temperature choose u_0 and T_0 according to problem

here we simply assume $u_0 = T_0 = 0$

Energy of a porous medium



Two phase system: $p \in [f, r]$

ϕ_p = vol. fraction of phase p

$m_p = \rho_p V_p$ mass of phase p

ρ_p = density of phase p

$V_p = \phi_p V$ volume of phase p

Total volume: $V = V_r + V_f$

$\phi = \phi_f$ porosity $\qquad \phi_r = (1-\phi)$

Internal energy of rock:

$$U_r = u_r m_r = u_r \rho_r V_r = u_r \rho_r \phi_r \underline{\underline{V}} = \int_\Omega \phi_r \rho_r c_{p,r} T_r \, dV$$

$$= (1-\phi) \rho_r c_{p,r} T_r$$

Internal energy of fluid:

$$U_f = u_f m_f = \int_\Omega \phi \rho_f c_{p,f} T_f \, dV$$

Total internal energy of porous medium:

$$U_T = U_f + U_r = \int_\Omega \left(\phi \rho_f c_{p,f} T_f + (1-\phi) \rho_r c_{p,r} T_r \right) dV$$

Assume local thermal equilibrium : $T_r = T_f = T$

$$U_T = \int_\Omega \underbrace{[\phi \rho_f c_{p,f} + (1-\phi)\rho_r c_{p,r}]}_{\overline{\rho c_p} = \phi \rho_f c_{p,f} + (1-\phi)\rho_r c_{p,r}} T \, dV$$

$$U_T = \int_\Omega \underbrace{\overline{\rho c_p} T}_{e} \, dV$$

$e =$ total energy density of porous medium per unit volume $\left[\frac{J}{m^3} = \frac{M}{LT^2}\right]$

Energy balance equation

General balance equation: $\frac{\partial u}{\partial t} + \nabla \cdot \underline{j} = \hat{f}_s$

$u =$ unknown $\quad \underline{j} =$ flux $\quad \hat{f}_s =$ source term

1) Unknown

$$\boxed{e = \overline{\rho c_p} T}$$

2) Energy fluxes

a) Conductive heat flux

Fourier's law: $\dot{\underline{j}}_c = -\kappa \nabla T$

where κ = thermal conductivity $\left[\frac{W}{mK} = \frac{ML}{T^3 \Theta}\right]$

This applies to each phase:

$\dot{\underline{j}}_{c,f} = -\kappa_f \nabla T \qquad \dot{\underline{j}}_{c,r} = -\kappa_r \nabla T$

Total conductive flux:

assume: $\dot{\underline{j}}_c = \phi\, \dot{\underline{j}}_{c,f} + (1-\phi)\, \dot{\underline{j}}_{c,r}$

$$= -(\phi \kappa_f + (1-\phi)\kappa_r)\, \nabla T$$

$$\boxed{\dot{\underline{j}}_c = -\bar{\kappa}\, \nabla T} \qquad \bar{\kappa} = \phi \kappa_f + (1-\phi)\kappa_r$$

b) advective heat flux

$\dot{\underline{j}}_A = \underline{v}\, \underbrace{\rho u}_{e} = \underline{v}\, \rho\, c_p\, T$ for single phase

applies to each phase

$\dot{\underline{j}}_{A,f} = \underline{v}_f\, \rho_f\, c_{p,f}\, T \qquad \dot{\underline{j}}_{A,r} = \underline{v}_r\, \rho_r\, c_{p,r}\, T$

Total adv. heat flux

$\dot{\underline{j}}_A = \phi\, \dot{\underline{j}}_{A,f} + (1-\phi)\, \dot{\underline{j}}_{A,r}$

$$= [\underbrace{\phi \underline{v}_f}_{\underline{q}} \rho_f c_{p,f} + (1-\phi) \cancel{\underline{v}_s}^{0} \rho_s c_{p,s}] T$$

$$\boxed{\underline{j}_A = \underline{q} \rho_f c_{p,f} T}$$

3) Source/sink $\hat{f}_s = 0$

Substitute into general balance law

$$\boxed{\overline{\rho c_p} \frac{\partial T}{\partial t} + \nabla \cdot [\underline{q} \rho_f c_{p,f} T - \bar{\kappa} \nabla T] = 0}$$

Transport Problem

if $\overline{\rho c_p} = \text{const}$ $(\phi = \text{const})$

$$- \nabla \cdot [\bar{\alpha} \nabla T] = 0$$

$$\Rightarrow \boxed{\frac{\partial T}{\partial t} + \nabla \cdot [\underline{v}_e T - \bar{\alpha} \nabla T] = 0}$$

standard form of Advection-Diffusion Eqn (ADE)

$$\bar{\alpha} = \frac{\bar{\kappa}}{\overline{\rho c_p}}$$ mean thermal diffusivity

$$\underline{v}_e = \underline{v}_f \underbrace{\frac{\phi \rho_f c_{p,f}}{\overline{\rho c_p}}}_{\leq 1}$$ effective velocity of the thermal front, which is

always less than fluid velocity
due to heat exchange with rock.