# Lecture 24: Gravity

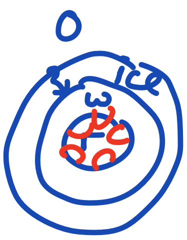
Logistics: - HW 10 due

- HW 11 $\rightarrow$ ADE in 2D

Last time: - Convection in porous media

mass: $-\nabla\cdot\left[\frac{k}{\mu_f}\nabla p\right] = f_s + \nabla\cdot\left[\frac{k}{\mu}\rho_f g \hat{z}\right]$

energy: $\overline{\rho c_p}\frac{\partial T}{\partial t} + \nabla\cdot\left[\mathbf{q}\rho_f c_{pf} T - \bar{\kappa}\nabla T\right] = 0$




constitutive laws:

1) $\mathbf{q} = -\frac{k}{\mu_f}\left(\nabla p + \rho_f g \hat{z}\right)$

2) $\rho_f = \rho_0\left(1 - \alpha(T - T_0)\right)$

$\Rightarrow$ gravity 'g' occurs in our equs

Today: - Gravity

- Solve "pressure equ" with gravity

# Gravity

gravitational field : $\underline{g} = -\nabla \Phi$

$\Phi$ is gravitational potential (scaler)

Gauss law of gravity

$$\boxed{\nabla \cdot \underline{g} = -4\pi G \rho}$$

$G$ = gravitational constant

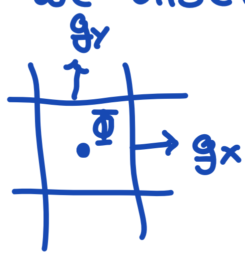
combine to get Poisson's equ for gravity

$$\boxed{\nabla^2 \Phi = 4\pi G \rho}$$

Similarity to "head formulation" of single phase porous flow:

$$\underline{g} \sim q$$
$$\Phi \sim h$$

If we discretize




$$\underline{g} = \begin{bmatrix} g_x \\ g_y \end{bmatrix}$$

$$\underbrace{\underline{\underline{D}} * \underline{\underline{G}}}_{\underline{\underline{L}}} * \underline{\Phi} = \underbrace{4\pi G \underline{\rho}}_{\underline{f_s}}$$

$$\underline{\underline{L}}\, \underline{\Phi} = \underline{f_s}$$

$\Rightarrow$ $\underline{g}$ is a quantity that lives on cell faces

# Gravity term in Darcy's law

$$q = -\frac{k}{\mu_f}\left(\nabla p + \rho_f g \hat{z}\right)$$

where $g = |\mathbf{g}|$

reformulate Darcy in terms of $\mathbf{g}$ vector

$$\boxed{\underset{\text{vector}}{\mathbf{g}} = -\underset{\text{scalar}}{g}\,\nabla z}$$

Note: minus sign because $\mathbf{g}$ points downward and $\nabla z$ points upward

Darcy: $$\boxed{q = -\underbrace{\frac{k}{\mu_f}}_{\lambda}\left(\nabla p - \rho_f \mathbf{g}\right)}$$

substitute into mass balance

$$\nabla \cdot q = f_s$$

$$\boxed{-\nabla \cdot \left[\frac{k}{\mu_f}\nabla p\right] = f_s - \nabla \cdot \left[\frac{k}{\mu_f}\rho_f \mathbf{g}\right]}$$

$k$ = permeability $\rightarrow$ $x_c$

$\mu_f$ & $\rho_f$ fluid properties $\rightarrow$ $x_c$

$\lambda = k/\mu_f$ "mobility" $\rightarrow$ $x_c$

$g$ = grav. field vector $\rightarrow$ $x_f$

Discretize l.h.s as before

$$-\nabla\cdot[\lambda \nabla p] \approx -\underline{\underline{D}} * \underset{\underset{\underline{\underline{Kd}}}{\uparrow}}{\underline{\underline{Lam}}} * \underline{\underline{G}} * \underline{p} = \underline{\underline{L}}\,\underline{p}$$

$$\underline{\underline{L}} = -\underline{\underline{D}} * \underline{\underline{Lam}} * \underline{\underline{G}}$$

by analogy to $\underline{\underline{Kd}}$

$$\underline{\underline{Lam}} = \text{comp\_mean}(\underline{Lam}, \underline{\underline{M}}, -1, Grid, 1)$$

Discretize the r.h.s.

$$f_s - \underbrace{\nabla\cdot[\lambda \rho_f g]}_{f_g} = f_s + f_g \approx \underline{f_s} + \underline{f_g}$$

$\Rightarrow$ new rhs vectors

New variables: $g$ = grav

$\underset{\sim}{g}$ = grav_vec

Continuous: $\underset{\sim}{g} = -g \nabla z$

Discrete: grav_vec = - grav * $\underline{\underline{G}}$ * Grid.xc (1D)

= - grav * $\underline{\underline{G}}$ * Yc(:)

↑ meshgrid

Note: grav_vec is zero on bnd's

due to natural BC's in $\underline{\underline{G}}$

$$f_g = - \nabla \cdot [\lambda \rho_f \underset{\sim}{g}]$$

$$\boxed{\underline{f_g} = - \underline{\underline{D}} * [\underline{\underline{Lam}} * \underline{\underline{Rho}} * \underline{\text{grav\_vec}}]}$$

What is the appropriate way to average $\rho$?

Not trivially obvious!

Rho = comp_mean($\underline{\text{rho}}$, $\underline{\underline{M}}$, 1, Grid, 1)

↑

debatable

# Flux Boundary conditions

head formulation: $\underset{\sim}{q} = -K\nabla h$

pressure form.: $\underset{\sim}{q} = -\frac{k}{\mu}(\nabla p - \rho \underset{\sim}{g})$

We want to prescribe physically meaning full net flux across bnd

$$q_B = \underset{\sim}{q} \cdot \underset{\sim}{n} \quad \text{rather than} \quad \nabla p \cdot \underset{\sim}{n} = ?$$

Split

$$\underset{\sim}{q} = \underset{\sim}{q}_p + \underset{\sim}{q}_g$$

$$\underset{\sim}{q} \cdot \hat{\underset{\sim}{n}} = (\underset{\sim}{q}_p + \underset{\sim}{q}_g) \cdot \hat{n} =$$

$$= \underset{\sim}{q}_p \cdot \hat{n} + \underset{\sim}{q}_g \cdot \hat{n}$$

$$\underset{\sim}{q}_p \cdot \hat{n} = \underset{\sim}{q} \cdot \underset{\sim}{n} - \underset{\sim}{q}_g \cdot \hat{n}$$

$$= q_B - \underset{\sim}{q}_g \cdot \hat{n}$$

$$\boxed{\underset{\sim}{q}_p \cdot \hat{n} = q_B - \underset{\sim}{q}_g \cdot \hat{n}}$$

need to specify

want to specif

subtract grav. flux

Remember how we compute fluxes

1) $q = -\underline{Lam} * (\underline{G} * p - \underline{Rho} * \underline{grav\_vec})$

works in interior

$q = 0$ on boundary

2) need to reconstruct flux

$q_b A = f_n V$

In build_bnd.m $\rightarrow f_n$

$f_n = q_b \frac{A}{V}$ without gravity

$$f_n = (q_b - q_g) \frac{A}{V}$$

$\uparrow$

subtract gravity flux