Lecture 25: Solving Convection

Logistics: - No more HW's
- Find out when grades are due
- Please fill out course evals

Last time: - Gravity

$$g = -\nabla \Phi$$
$$\nabla \cdot g = -4\pi G \rho$$
$$\nabla^2 \Phi = 4\pi G \rho$$

$\Rightarrow$ $g \rightarrow$ flux $\rightarrow$ on faces

- Darcy's law with gravity

$$q = -\frac{k}{\mu_f}(\nabla p + \rho g \hat{z}) \qquad g = |g|$$
$$= -\frac{k}{\mu_f}(\nabla p - \rho_f \bar{g})$$
$$q = q_p + q_g$$
$$\underset{\uparrow\atop \underline{\underline{\text{kd}}}}{-\nabla \cdot \left(\frac{k}{\mu} \nabla p\right)} = f_s \underbrace{- \nabla \cdot \left[\frac{k}{\mu_f} \rho_f g\right]}_{\uparrow\ \underline{\text{kd}}}$$
$$= f_s + f_g$$

$\underline{\underline{\text{Rhod}}}$ = comp_mean(rho, M, 1, Grid, 1)

- build_bnd_grav

$q \cdot \hat{n} = q_b$

$q_p = q - q_g$

Today: Numerical soln for Convection

- Numerical Soln for ADE with variable co. properties
- Coupling of flow & transport

Energy conservation:

$$\overline{\rho c_p} \frac{\partial T}{\partial t} + \nabla \cdot [q \, \rho_f c_{p,f} \, T - \underline{\bar{\kappa}} \nabla T] = 0$$

$\underline{\underline{A}}(q)$ $N_f \cdot N$ $\quad$ $\underline{u}$ $N \cdot 1$

$$\overline{\rho c_p} = \phi \rho_f c_{p,f} + (1-\phi) \rho_r c_{p,r}$$

rho_cp_mean = phi * rho_f * cp_f + (1-phi).* rho_r.* cp_r

$\underline{\underline{S}}$ = spdiags(rho_cp_mean, 0, N, N)

kappa_mean = phi * kappa_f + (1-phi).* kappa_r

$\underline{\underline{Kd}}$ = comp_mean(kappa_mean, $\underline{\underline{M}}$, -1, Grid, 1)

$\underline{\underline{RhoCp}}$ = spdiags(rhof * cp_f, 0, N, N),

Full discretization:

$$\underline{\underline{S}}\left(\underline{u}^{n+1}-\underline{u}^{n}\right) + \Delta t * \underbrace{\underline{\underline{D}} * \left(\underline{\underline{A}}(q)\,\underline{\underline{RhoCp}} - \underline{\underline{Kd}}\,\underline{\underline{G}}\right)}_{\underline{\underline{L}}}\left(\theta \underline{u}^{n} + (1-\theta)\underline{u}^{n+1}\right)$$

$$\underline{\underline{IM}} = \underline{\underline{S}} + \Delta t\,(1-\theta)\,\underline{\underline{L}}$$

$$\underline{\underline{EX}} = \underline{\underline{S}} - \Delta t\,\theta\,\underline{\underline{L}}$$

Coupling Flow and Transport

Each problem by itself is linear, only the coupling is non-linear! ⇒ typical of convection problems

System of PDE's:

mass: $$-\nabla\cdot\left[\frac{k}{\mu_f}\nabla p\right] = f_s - \nabla\cdot\left[\frac{k}{\mu_f}\rho_f(T)\,g\right]$$

energy: $$\overline{\rho c_p}\,\frac{\partial T}{\partial t} + \nabla\cdot\left[q(p)\,\rho_f\,c_{p,f}\,T - \overline{\kappa}\,\nabla T\right] = 0$$

const. laws: 1) $$q = -\frac{k}{\mu}\left(\nabla p - \rho_f(T)\,g\right)$$

2) $$\rho_f = \rho_0\left(1 - \alpha\,(T - T_0)\right)$$

Strictly we have to solve both equations simultaneously

⇒ non-linear algebraic system

use Newtons method.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \underline{p} \\ \underline{u} \end{bmatrix}^{n+1} = rhs$$

this is most proper way

but it is expensive multiple linear solves per timestep

⇒ Fully coupled / Fully Implicit solution

Instead we simply lag solution

→ IC for T → p

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for i = 1 : Nt
    [p,q] = solve_flow (T);
    T = solve_trans (q) ;
end
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