

Kronecker product assembly of advection matrices

Problem: In $\underline{\underline{D}}$ and $\underline{\underline{G}}$ the matrix blocks are identical, but in $\underline{\underline{A}}$ each block has same structure but values differ because q 's vary across the domain.

Solution: Separate the structure, i.e. the 1's and 0's from the magnitudes.

The overall scheme for computing advective fluxes $\underline{\underline{a}}$:

$$\begin{matrix} 1 \\ \vdots \\ x \\ \vdots \\ y \\ \vdots \\ q \end{matrix} = \left(\begin{matrix} N_f & & & \\ & \underline{\underline{Qd}}^+ & & \\ & & N & \\ & & & \underline{\underline{A}}^+ \\ & & & & N_f & & & \\ & & & & & \underline{\underline{Qd}}^- & & \\ & & & & & & N & \\ & & & & & & & \underline{\underline{A}}^- \end{matrix} \right) \begin{matrix} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{matrix}$$

Where we have following sparse matrices:

- $\underline{\underline{Qd}}^+$ = N_f by N_f matrix with positive fluxes on diagonal
 - $\underline{\underline{Qd}}^-$ = N_f by N_f matrix with negative fluxes on diagonal
 - $\underline{\underline{A}}^+$ = N_f by N matrix with ones in location of pos. fluxes
 - $\underline{\underline{A}}^-$ = N_f by N matrix with ones in location of neg. fluxes
- } magnitudes
} structure

If flow is evolving only $\underline{\underline{Qd}}^+$ and $\underline{\underline{Qd}}^-$ must be updated!

$$Qdp = \text{spdiags}(\max(q, 0), 0, N_f, N_f);$$

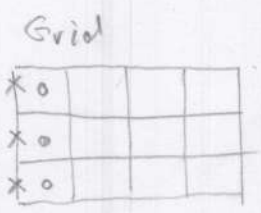
$$Qdn = \text{spdiags}(\min(q, 0), 0, N_f, N_f);$$

So that: $\underline{\underline{A}}(q) = \underline{\underline{Qdp}}(q) * \underline{\underline{A}}_p + \underline{\underline{Qdn}}(q) * \underline{\underline{A}}_n$

Assembly of $\underline{A}x^+$, $\underline{A}x^-$, $\underline{A}y^+$ and $\underline{A}y^-$ with Kronecker products

Ax - matrices

A_x computes N_y by (N_x+1) fluxes from N_y by N_x concentration

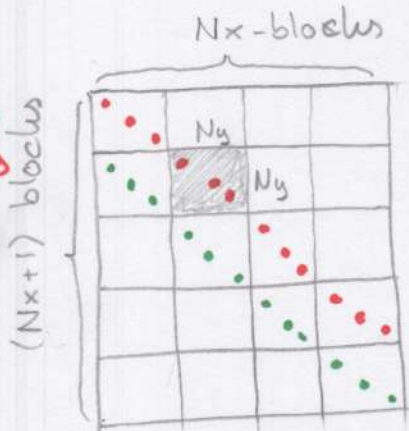


N_x columns of N_y concentration

N_x+1 columns of N_y fluxes

Each block is N_y by N_y

• pos. fluxes
• neg. fluxes



$$\underline{I}_y = \text{speye}(N_y);$$

Positive fluxes are on -1 block diagonal

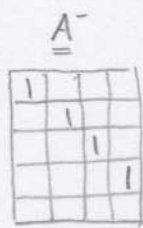
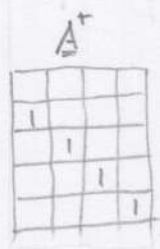
ID \underline{A}^+ matrix:

$$A_{xp1} = \text{spdiags}(\text{ones}(N_x, 1), -1, N_x+1, N_x);$$

ID \underline{A}^- matrix:

$$A_{xn1} = \text{spdiags}(\text{ones}(N_x, 1), 0, N_x+1, N_x);$$

ID matrices:

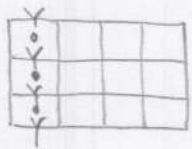


2D matrices: $A_{xp} = \text{kron}(A_{xp1}, I_y);$

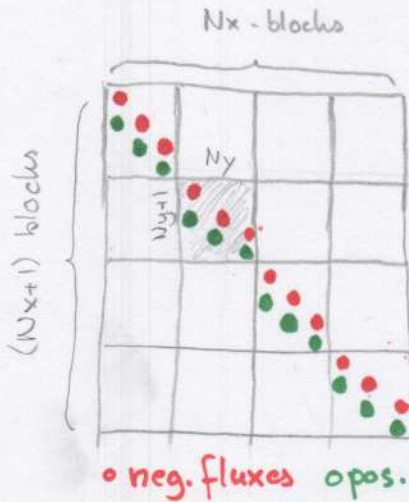
$A_{xn} = \text{kron}(A_{xn1}, I_y);$

Ay - matrices

Ay computes Nx columns of Ny+1 fluxes from Nx columns of Ny concentrations



⇒ Ay is Nx by Nx block matrix with blocks of size Ny+1 by Ny



Overall block structure

$$\underline{I_x} = \text{speye}(N_x);$$

Each block is ID Matrix:

$$A_{xp1} = \text{spdiags}(\text{ones}(N_y, 1), -1, N_y+1, N_y);$$

$$A_{xn1} = \text{spdiags}(\text{ones}(N_y, 1), 0, N_y+1, N_y);$$

Assemble 2D y-matrices: $A_{yp} = \text{kron}(I_x, A_{yp1});$

$$A_{yn} = \text{kron}(I_x, A_{yn1});$$

Assemble overall 2D matrices:

$$\underline{A_p} = \begin{bmatrix} \underline{A_{xp}} \\ \underline{A_{yp}} \end{bmatrix}$$

$$\underline{A_n} = \begin{bmatrix} \underline{A_{xn}} \\ \underline{A_{yn}} \end{bmatrix}$$