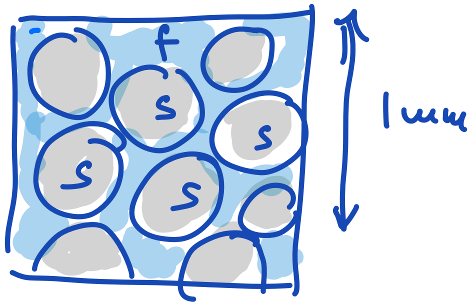
Intro to porous media



Saturated porous medium

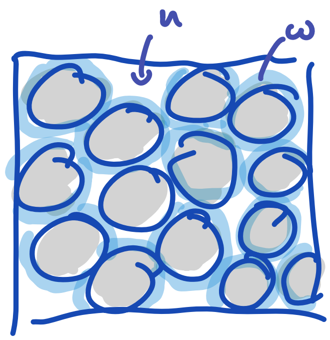
Two phases:

1) Solid (s)

2) pore fluid (f)

$\Rightarrow$ calculating fluxes of pore fluid

linear



Unsaturated porous medium

Three phases

1) Solid (s)

2) wetting fluid (w) – water

3) non-wetting fluid (n) – air

non-linear

Volume fraction: $\boxed{\phi_p = \frac{V_p}{V_T}}$ $\quad 0 \le \phi_p \le 1$

V_p = volume of phase $p \in \{w, n, s\}$

$V_T = \sum_p V_p$ total volume

$\Rightarrow \boxed{\sum_p \phi_p = 1}$ volume fraction constraint

Porosity: $\phi = \phi_f$ (saturated)

$\phi = \phi_w + \phi_n$ ~~we~~ (unsaturated)

Fluid saturations: $\boxed{s_p = \phi_p / \phi}$ $p \in [w, n]$

$$\sum_p s_p = 1$$

s_p is fraction of pore space occupied by fluid p.

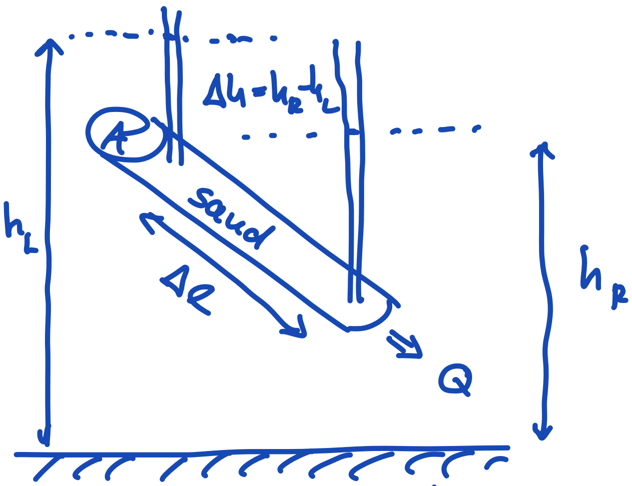
$s_w = 1 \Rightarrow$ saturated medium

Moisture content: $\theta = \phi_w$

D

Darcy's law

Q = volumetric flow rate $\left[\frac{L^3}{T}\right]$



A = cross-sectional are $[L^2]$

h_L, h_R = water elevations in manometers (hydraulic heads) $[L]$

$\Delta h = h_R - h_L \quad [L]$

ΔL = distance between manometer

Exerimental observations

1) $Q \sim -\Delta h$
2) $Q \sim \frac{1}{\Delta l}$
3) $Q \sim A$

$$Q \sim -A \frac{\Delta h}{\Delta l}$$

$\Rightarrow$ Darcy's law

$$\boxed{Q = -K A \frac{\Delta h}{\Delta L}}$$

$$\frac{L^3}{T} \qquad \frac{L}{T} \quad L^2 \quad \frac{L}{L}$$

Hydraulic conductivity: $K \quad \left[\frac{L}{T}\right]$

constant of proportionality

Comments: 1) Empirical law (o.k.)
2) Macroscopic (good)
3) Q is an integrated quantity
it depends on A

For continuum theories we need fluxes not rates!

Rate: amount of something per time $\left[\frac{\#}{T}\right]$
discharge: Q $\frac{L^3}{T}$ $\rightarrow$ scalar

Flux: amount of something per area per time
$\left[\frac{\#}{L^2 T}\right]$ $\longrightarrow$ vector
specific discharge: $\underline{q} = \frac{Q}{A}\hat{n}_A$ $\left[\frac{L^3}{L^2 T} = \frac{L}{T}\right]$
Note: $\underline{q}$ is not flow velocity
$\underline{v} = \frac{\underline{q}}{\phi}$ ave. flow velocity

in 1D: $|\underline{q}| = -K \frac{\Delta h}{\Delta L}$
3D: $\underline{q} = -K \nabla h$ ∇h = gradient