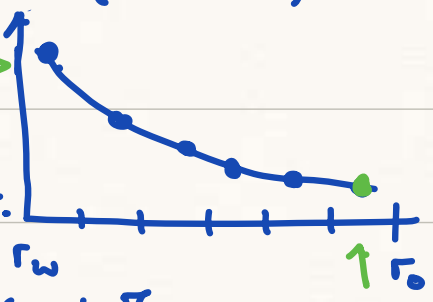
Lecture 14: Numerical Streamfunction

Logistics: - HW4 is due 4 (10/17)

Problem 3: $q_w \rightarrow$



Dir. BC at cell cent.

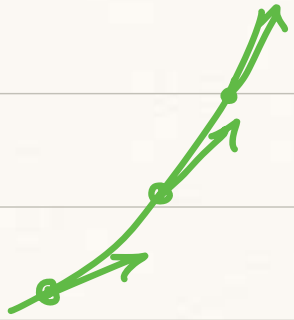
Neu. BC at face (r_w) !

if you submit by end of day not late !

- HW2 is complete ✓
- HW5 posted due 03/07

⇒ 2D operators

Last time: - Stream lines



curves tangent to velocity/flux

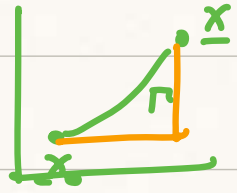
- Stream function

$$\psi(\underline{x}) = \psi_o(x_o) - \int_{x_o}^{x} q_y\,dx + \int_{y_o}^{y} q_x\,dy$$

$$\frac{d\underline{x}}{dt} = \underline{v} = \frac{\underline{q}}{\phi}$$

FTC: $\frac{\partial \psi}{\partial x} = -q_y$, $\frac{\partial \psi}{\partial y} = q_x$

$\nabla \cdot q = 0$

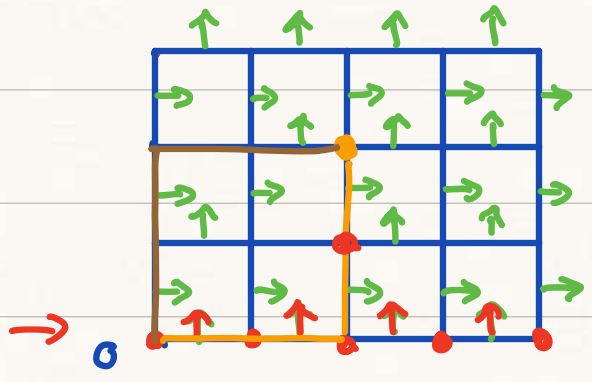


⇒ $\nabla\psi \perp q$ and $|\nabla\psi| = |q|$

Today: Compute streamfunction

Computing Stream function

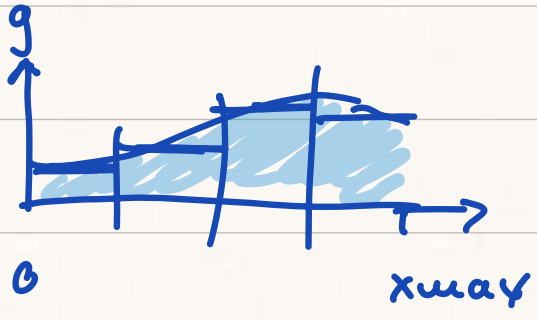
Definition: $\psi(x,y) = \psi_0(x_0,y_0) - \int_{x_0}^{x} q_y\,dx + \int_{y_0}^{y} q_x\,dy$



Nx = 4 Ny = 3

$\int_0^x q_y(x,0)\,dx \approx$

q(Grid.dof-ymin) *Grid.dx



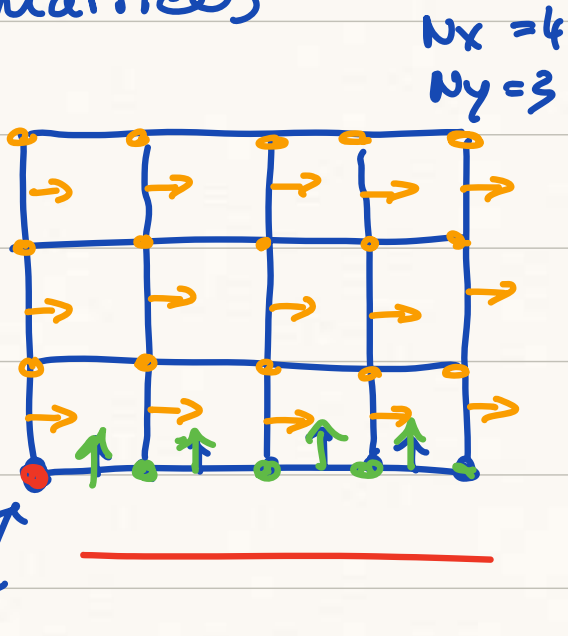
Given location of flux components along faces the natural location for integral/streamfunction is the cell corners.

Hints on implementation

- Simple Riemann sum to evaluate integral best done using cumsum.m

- cumsum also works on matrices
- Two step process:

1) cumsum along bnd (ymin)



$[\psi_0; \text{cumsum}(q(\text{Grid.dof}-y\text{min}) \cdot dx)]$ $\underline{x_0}$

N_x+1

ref

2) Integrate upward into domain by applying cumsum to appropriately reshaped matrix of qx velocities

```
qx = q(1 : Nfx)
Qx = reshape(qx, Ny, Nx+1);
cumsum(Qx, )
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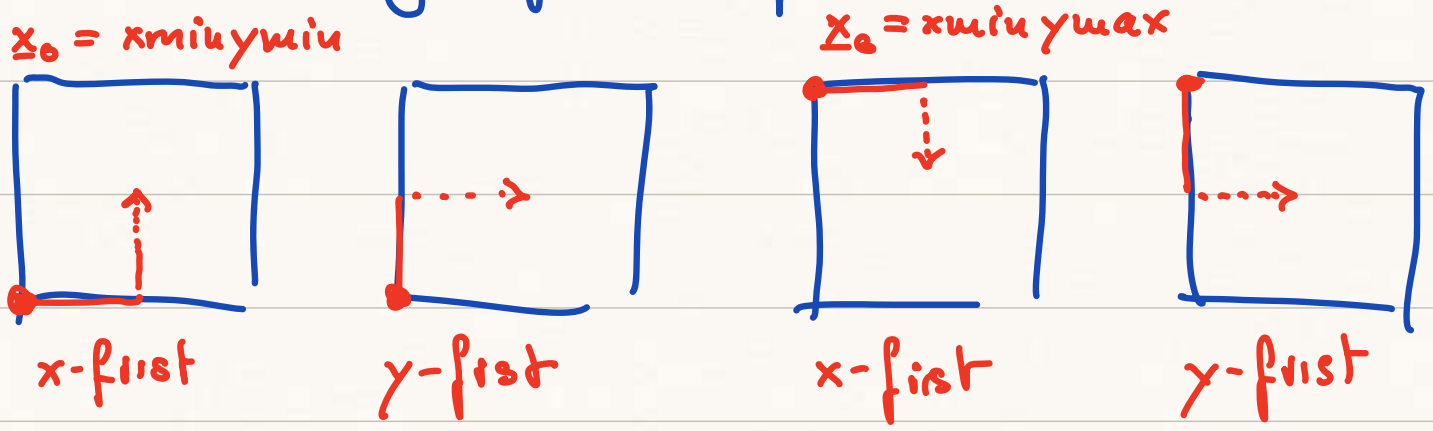
Need to add vector of initial values alongx base

Note on path independence:

ψ is uniquely defined (up to a constant)



The choice of starting point affects the constant ψ_0, but choice of integration path does not.

Once Psi is known you just plot contours.