

Conservative Source Term

Aim: Discrete Conservation

$$\text{PDE: } -\nabla \cdot \kappa \nabla T = \rho H_0 e^{-\frac{z}{h_r}}$$

$$\text{BC: } T(0) = T_s \quad \mathbf{q} \cdot \hat{\mathbf{z}} = -|q_w|$$

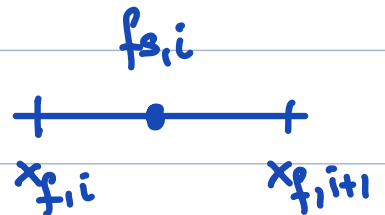
$$\text{Integrating: } q_s = \int_0^h \rho H_0 e^{-\frac{z}{h_r}} dz - |q_w|$$

$$q_s = + |q_w| + \rho H_0 h_r (1 - e^{-\frac{h}{h_r}}) > 0$$

If energy is conserved discretely and q_w is implemented correctly $\Rightarrow q_s$ should be exact!

\Rightarrow any errors due to source term

Average source term correctly:



$$\langle f_{s,i} \rangle = \frac{1}{\Delta x} \int_{x_{f,i}}^{x_{f,i+1}} \rho H_0 e^{-\frac{z}{h_r}} dx = \frac{\rho H_0 h_r}{\Delta x} \left(e^{-\frac{x_{f,i}}{h_r}} - e^{-\frac{x_{f,i+1}}{h_r}} \right)$$