Energy Conservation Equation

Internal energy: Energy of a body not associated with kinetic or potential energy.

Internal energy ~ thermal energy / heat

symbol: E units: Joule $= \left[\frac{ML^2}{T^2}\right]$

specific internal energy / energy density

$e = \frac{E}{m}$ $m = \text{mass}$ $\left[\frac{L^2}{T^2} = \frac{J}{kg}\right]$

$\boxed{de = c_p \, dT}$ $T = \text{temperature}$

$c_p =$ specific heat capacity at const. pressure $\left[\frac{J}{kg\,K} = \frac{L^2}{T^2 \Theta}\right]$

Physical interpretation:

c_p is the heat required to raise the temerature of 1 kg by 1 degree K.

Energy density: $\boxed{e(T) = e_0 + c_p (T - T_0)}$

e_0 = ref. energy

T_0 = ref. temperature

$\left.\begin{matrix} e_0 \\ T_0 \end{matrix}\right\}$ const.

General balance equation: $\frac{\partial u}{\partial t} + \nabla \cdot \underline{j} = \hat{f}_s$

u = unknown , $\underline{j}$ = flux , $\hat{f}_s$ = source/sink

1) Unknown: $u \; \left[\frac{\#}{L^3}\right]$

$\# \rightarrow$ energy $\Rightarrow u \; \left[\frac{J}{m^3}\right]$

but $e \; \left[\frac{J}{kg}\right]$

$\Rightarrow u = \rho e = \rho (e_0 + c_p (T - T_0))$

2) Energy fluxes $\Rightarrow$ conductive

Fourier's law: $\underline{j} = - \kappa \nabla T$

whre κ = thermal conductivity

units $\left[\frac{W}{mK} = \frac{ML}{T^3 \Theta}\right]$

3) Source/Sink $\hat{f}_s = 0$

because e is a conserved quantity

Substitute into the general balance law

$$\boxed{\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot [k \nabla T]}$$ Heat equation

If ρc_p = const.

$$\Rightarrow \quad \frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

$$\alpha = \frac{k}{\rho c_p}$$ thermal diffusivity $\left[\frac{L^2}{T}\right]$