Energy Conservation Equation

Internal energy: Energy of a body not associated with kinetic or potential energy.

Internal energy ~ thermal energy/ heat

symbol: E units: Joule $= \left[\frac{ML^2}{T^2}\right]$

specific internal energy /energy density

$e = \frac{E}{m}$ $m =$ mass $\left[\frac{L^2}{T^2} = \frac{J}{kg}\right]$

$\boxed{de = c_p \, dT}$ $T =$ temperature

$c_p =$ specific heat capacity at const. pressure $\left[\frac{J}{kg\,K} = \frac{L^2}{T^2 \Theta}\right]$

Physical interpretation:

c_p is the heat required to raise the temerature of 1 kg by 1 degree K.

Energy density: $\boxed{e(T) = e_0 + c_p(T - T_0)}$

$\left.\begin{array}{l} e_0 = \text{ref. energy} \\ T_0 = \text{ref. temperature} \end{array}\right\}$ const.

General balance equation: $\frac{\partial u}{\partial t} + \nabla \cdot \underline{j} = \hat{f}_s$

u = unknown, $\underline{j}$ = flux, $\hat{f}_s$ = source/sink

1) Unknown: $u \left[\frac{\#}{L^3}\right]$

$\# \to$ energy $\Rightarrow u \left[\frac{J}{m^3}\right]$

but $e \left[\frac{J}{kg}\right]$

$\Rightarrow u = \rho e = \rho(e_0 + c_p(T - T_0))$

2) Energy fluxes $\Rightarrow$ conductive

Fourier's law: $\underline{j} = -\kappa \nabla T$

where κ = thermal conductivity

units $\left[\frac{W}{mK} = \frac{ML}{T^3 \Theta}\right]$

3) Source/Sink

many kinds:
- radiogenic decay heating
- adiabatic cooling
- heats of reactions
- latent heat phase change
- Friction (earthquakes)

For now standard radiogenic source:

$$\hat{f}_s = \rho H_0$$

H_0 = heat production per unit mass $\left[\frac{W}{kg} = \frac{J}{kg\,s}\right]$

Substitute into the general balance law

$$\boxed{\rho c_p \frac{\partial T}{\partial t} - \nabla \cdot [k \nabla T] = \rho H_0}$$

Heat equation

If ρc_p = const. $\Rightarrow$ $\frac{\partial T}{\partial t} - \alpha \nabla^2 T = \frac{H_0}{c_p}$

$\alpha = \frac{k}{\rho c_p}$ thermal diffusivity $\left[\frac{L^2}{T}\right]$