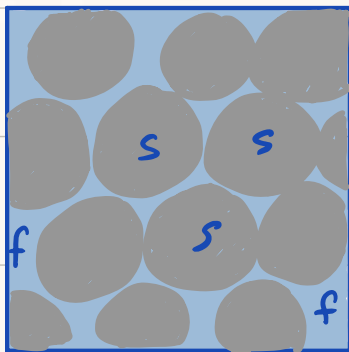


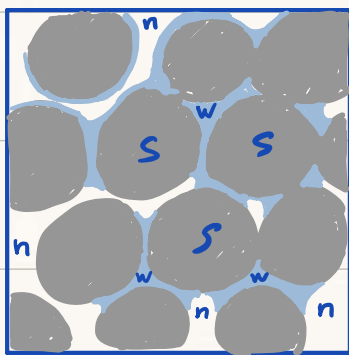
Introduction to porous media



A saturated porous medium

comprises two phases:

- 1) solid (s)
- 2) pore-fluid (f)



An unsaturated porous medium

comprises three phases:

- 1) solid (s)
- 2) wetting fluid (w) - water
- 3) non-wetting fluid (n) - air

Volume fractions:

$$\phi_p = V_p / V_T$$

V_p = volume of phase p

$V_T = \sum_p V_p$ = total volume

$$\Rightarrow \sum_p \phi_p = 1 \quad \text{vol. frac. constraint}$$

Porosity: $\phi = \phi_f$ (saturated)

$$\phi = \phi_w + \phi_n \text{ (unsaturated)}$$

ϕ is the fraction of space occupied by pores!

Fluid saturations: $s_p = \phi_p / \phi$ $p \in [w, n]$

$$\sum_p s_p = 1$$

s_p is the fraction of pore space occupied by fluid p .

$s_p = 0$ or $s_p = 1 \Rightarrow$ saturated medium

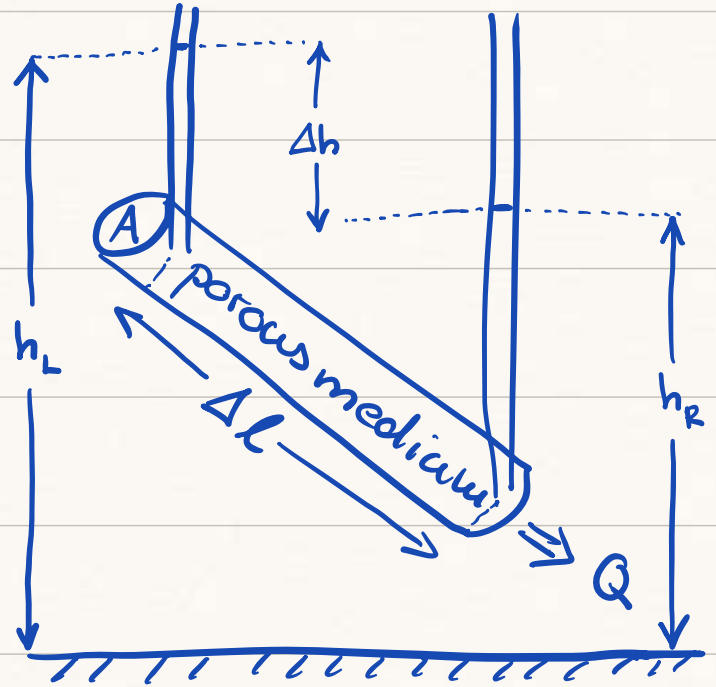
Darcy's law

Δl = sample length [L]

A = cross-section [L^2]

h_L, h_R = water elevations in
left & right manometers
 $\Rightarrow$ hydraulic heads [L]

$\Delta h = h_R - h_L$ = head change [L]



Q = volumetric flow rate/discharge [L^3/T]

Experimental observations

1) $Q \sim -\Delta h$

2) $Q \sim 1/\Delta l$

3) $Q \sim A$

$Q \sim -A \frac{\Delta h}{\Delta l}$

$\Rightarrow$ Darcy's law

$$Q = -K A \frac{\Delta h}{\Delta l}$$

Hydraulic conductivity: K [L/T]

(constant of proportionality)

Comments: 1) Empirical law (o.k.)

2) Macroscopic law (good)

3) Q is an integrated quantity it depends on A (bad)

For continuum theories we need fluxes not rates!

Rate: amount of something per time $[\#/T]$

example: discharge Q $[L^3/T] \rightarrow$ scalar

Flux: amount of something per area per time
 $[\#/(L^2 T)] \rightarrow$ vector

example: specific discharge $q = \frac{Q}{A} \hat{n}_A$ $[\frac{L^3}{L^2 T} = \frac{L}{T}]$

$\hat{n}_A$ = unit normal of A in dir. of flow

Note: q is not a velocity!

In 1D: $|q| = -K \frac{\Delta h}{\Delta l}$

Darcy's law for flux in 3D

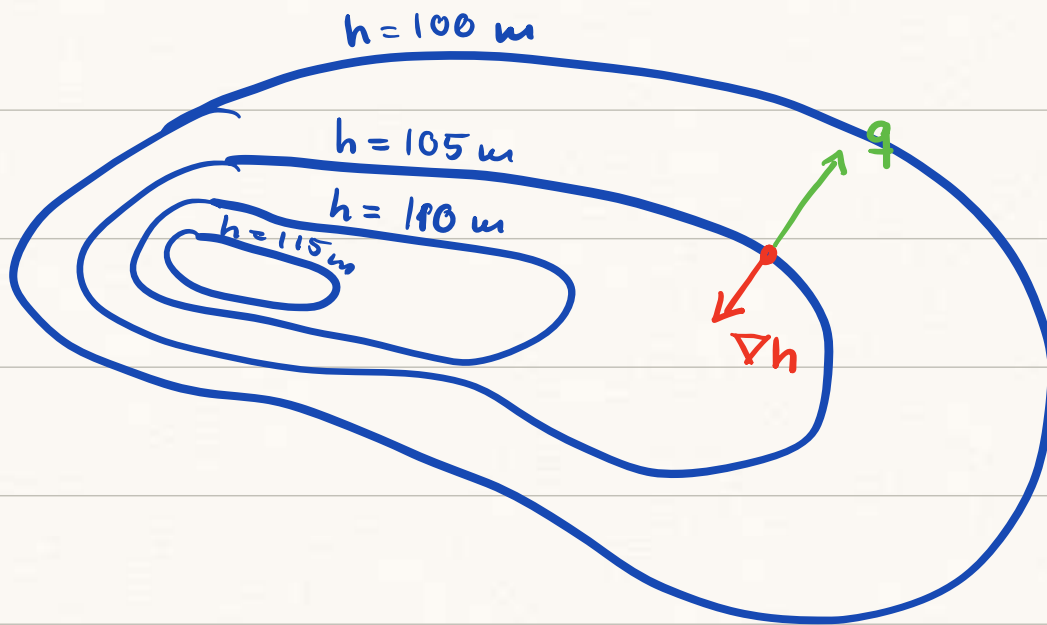
$$\underline{q} = - \underline{K} \nabla h$$

$$\underline{q} = \begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} \text{ flux vector}$$

$$\nabla h = \begin{pmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \\ \frac{\partial h}{\partial z} \end{pmatrix} \text{ gradient of the head}$$

$$\underline{K} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{xy} & K_{yy} & K_{yz} \\ K_{xz} & K_{yz} & K_{zz} \end{bmatrix} \text{ hyd. conductivity tensor } (\underline{K} = \underline{K}^T)$$

Given head distribution $h = h(x, y)$



Note: Gradient ∇h is always perpendicular to level sets of $h(\underline{x}) = h_0$.

Only if $\underline{K} = \underline{k}(\underline{x}) \underline{I}$ is isotropic then $\underline{q}$ is also perpendicular to level sets of h .

Heterogeneity & Anisotropy

The hydraulic conductivity K can depend both on location and direction?

Heterogeneity: $K = K(\underline{x})$ (dependence on location)

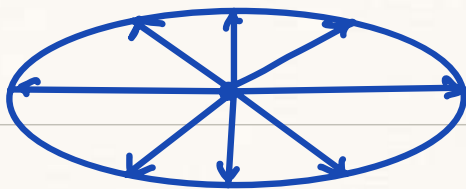
strongly dependent on rock type

6 - orders of magnitude variation

⇒ key to subsurface flow

key to this class

Anisotropy: $\underline{K}$ tensor/matrix (dependence on direction)



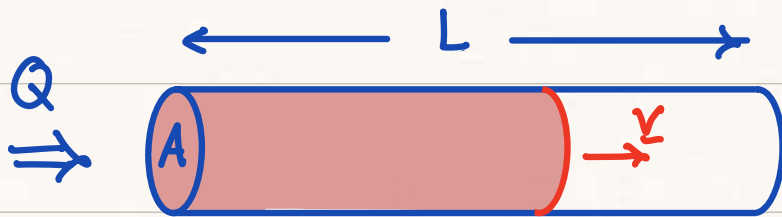
best visualized as
ellipse

⇒ numerically challenging.

Relationship between flux and velocity

It is important to realize that Darcy's law gives the volumetric flux, q , not the average interstitial fluid velocity, $\underline{v}$!
ave. interstitial velocity $\Rightarrow$ velocity of dye front

Consider injection of fluid with red dye into a saturated sand packed column:



Dye arrives at out let when the entire pore volume has been replaced.

Pore volume: $V_p = \phi A L$

Time required to inject one pore volume T :

$$V_p = Q T = A q T = A \phi v T$$

equating: $\phi A L = A \phi T v \Rightarrow \underline{v} = \frac{L}{T} \checkmark$