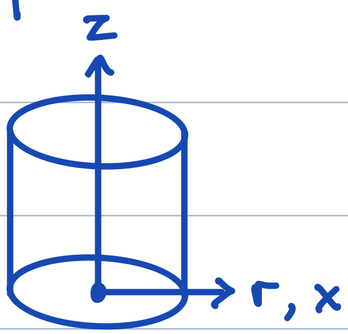
Non-Cartesian Coordinates (1D)

$\nabla\cdot, \nabla, \nabla\times$ notation hides coordinate system

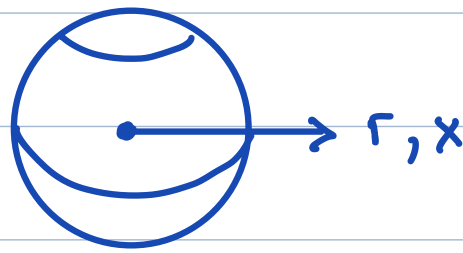
$\Rightarrow$ same for discrete ops: $\underline{\underline{D}}, \underline{\underline{G}}, \underline{\underline{C}}$

Even in one-direction 'x':

1) linear cartesian (1D)
2) cylindrical radial (2D) $\rightarrow$ wells
3) spherical radial (3D) $\rightarrow$ planets



cylindrical



spherical

Gradient is unchanged: $\nabla = \frac{d}{dx}$

Divergence changes:

linear (1D): $\nabla\cdot = \frac{d}{dx}$

cylindrical (2D): $\nabla\cdot = \frac{1}{x}\frac{d}{dx}x$

spherical (3D): $\nabla\cdot = \frac{1}{x^2}\frac{d}{dx}x^2$

Divergence in 'd' dimensions:

$$\boxed{\nabla\cdot = \frac{1}{x^{d-1}}\frac{d}{dx}x^{d-1}}$$

Geometric Interpretation:

⇒ key to correct discretization!

$$-\nabla\cdot[K\nabla h] = f_s$$

explicit in 'd' dimensions

$$-\frac{1}{x^{d-1}}\frac{d}{dx}\left[x^{d-1}K\frac{dh}{dx}\right] = f_s$$

$$-\frac{d}{dx}\left[\underbrace{x^{d-1}K}_{\text{flux}}\frac{dh}{dx}\right] = \underbrace{x^{d-1}f_s}_{\text{source}}$$

- $x^{d-1}K$ represents the growth of interfacial area with x
  ⇒ evaluate at faces (Grid.xf)
- $x^{d-1}f_s$ represents the growth of cell volume with x
  ⇒ evaluate at cell centers (Grid.xc)

Discretizing radial divergence

Continuous: $\nabla\cdot = \frac{1}{x^{d-1}} \frac{d}{dx}\, x^{d-1}$

Discrete: $\underline{\underline{D}}(d) = \underline{\underline{Rinv}}(d)\, \underline{\underline{D}}\, \underline{\underline{R}}(d)$

$$\underline{\underline{Rinv}} = \begin{bmatrix} \ddots & & \\ & \frac{1}{x_c^{d-1}} & \\ & & \ddots \end{bmatrix} \qquad \underline{\underline{R}} = \begin{bmatrix} \ddots & & \\ & x_f^{d-1} & \\ & & \ddots \end{bmatrix}$$

$$Nx \cdot Nx \qquad\qquad Nfx \cdot Nfx$$

Matlab:

```
Rinv = @(d) spdiags(1./Grid.xf.^(d-1),0,Nx,Nx)
R =@(d) spdiags(Grid.xc.^(d-1),0,Nfx,Nfx)
```

Test problem:

PDE: $-\nabla \cdot [K \nabla h] = 0 \qquad x \in [x_{min}, x_{max}] \qquad x_{min} > 0$

BC: $h(x_{min}) = h_1 \qquad h(x_{max}) = h_2$

linear: $$h(x) = h_1 + \frac{h_2 - h_1}{x_{max} - x_{min}} (x - x_{min})$$

cylindrical: $$h(x) = h_1 + (h_2 - h_1) \frac{\ln(x/x_{min})}{\ln(x_{max}/x_{min})}$$

spherical: $$h(x) = h_1 + \frac{h_2 - h_1}{\frac{1}{x_{max}} - \frac{1}{x_{min}}} \left(\frac{1}{x} - \frac{1}{x_{min}} \right)$$